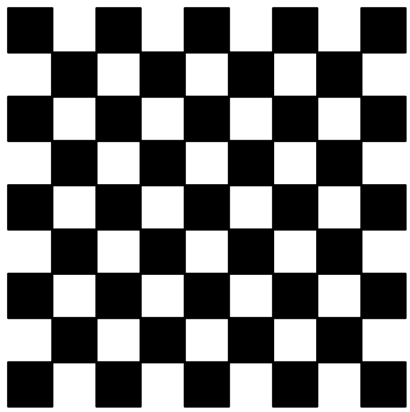
1. [4 points] We have an equilateral triangle TRI and a square $RAIL$ in the plane with T closer to L than to A. Let M be a point in the plane such that the area of $TRAM$ is $7 + 7\sqrt{3}$. Find the area of $ARML$.

2. [4 points] What is the largest integer n such that $\dfrac{n^2 - 2018}{n + 7}$ is an integer?

3. [4 points] Find the number of ordered pairs of nonzero digits A, B such that the 2-digit integer AB divides the 2-digit integer BA.

4. [4 points] Consider a $9 \times 9 \times 9$ cube made up of 729 unit cubes. We then remove unit cubes such that looking at every side of the cube we see a checkerboard:



where each white square represents a column of cubes that was removed, and each black square represents a column with at least one cube present. What is the maximum number of cubes in the final configuration?

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5. [6 points] Find $\cot^{-1} \frac{1}{2} - \cot^{-1} \frac{1}{3}$. Express your answer in radians.

6. [6 points] Let $3125! = m \cdot 5^k$, where m and k are positive integers. Find the number of ordered pairs (m, k) satisfying $m + k \equiv 0 \mod 5$.

7. [6 points] Find the number of ways that 10 can be written as the sum of 1's, 2's, and 3's, where the order of the sum matters. For example $1 + 3 + 3 + 3$ is different from $3 + 1 + 3 + 3$.

8. [6 points] A certain pyramid has a base that is a triangle with side lengths 13, 14, and 15. The other three edges of this triangle are pairwise perpendicular. What is the volume of this pyramid?

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9. [8 points] The set of points inside a non-degenerate triangle ABC that are closer to the incenter(the intersection of the angle-bisectors) than any of the vertices is a polygon with k sides. What is the sum of all possible values of k.

10. [8 points] Find the number of ordered pairs of integers (x, y) that satisfy $x^2 - xy + y^2 = x + y$.

11. [8 points] How many ways are there to fill a 2x5 grid with the numbers 1 through 10 such that each number is smaller than the ones adjacent to it to the left and above?

12. [8 points] Find the number of pairs of positive integers (x, y) such that $6x^4 + 2 = y^3$.

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13. [10 points] Let circle γ be a circle with radius 5 centered at point O, and let circle Γ be a circle with radius 7 also centered at point O. Let point A be on circle γ and points B and C on circle Γ so that $AB = 3$ and $AC = 8$. Find BC^2.

14. [10 points] Bill and Jinho start on opposite squares of a 2x2 grid. Every second they either move to one of the two adjacent squares, or stay on the same square. After 5 billion years the probability that Bill and Jinho are on opposite squares is n, what is $\lfloor 666n \rfloor$?

15. [10 points] How many divisors of 25! are perfect powers?(A perfect power is an integer where the GCF of the powers of its prime factors is greater than 1).

16. [10 points] What is the value of $\dfrac{1}{\sum\limits_{a>b>c>0}^{\infty} 2^{-a}3^{-b}5^{-c}}$?

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17. [12 points] A sequence of 7 positive integers is called "froggy" if, for each positive integer $k \geq 2$, if the number k appears in the sequence, then so does $k-1$, and the first occurrence of $k-1$ comes before the last occurrence of k. How many froggy sequences are there?

18. [13 points] For a certain positive integer x, x^2 is 2 less than a multiple of a prime number p, and x^3 is 9 less than a multiple of p. What is the sum of p and the smallest positive value for x?

19. [14 points] Let a, b, and c be the distinct complex roots of $x^3 + x^2 + x + 3 = 0$. Find the value of

$$\frac{a+b}{1-ab} + \frac{b+c}{1-bc} + \frac{a+c}{1-ac}$$

20. [15 points] There is a cyclic quadrilateral $JOHN$, points B, and L are on segment JN such that OB, and HL intersect on the circumcircle of $JOHN$. If $JB = 2$,$BL = 3$, $LN = 4$, and $OH = 5$, what is $JO \cdot HN$?