Team round 1

1. [2 points] Let $f(x) = \lfloor \sqrt{|x|} \rfloor + \sqrt{\lfloor |x| \rfloor}$, find the sum of all values of n such that $f(n) = 2018$.

2. [3 points] Let M be a point on the perimeter of triangle ABC such that $AM = 5$, $BM = 6$, and $CM = 7$. Find the maximum possible area of ABC.

3. [5 points] Let $ABCD$ be a cyclic quadrilateral with $AB = 6, BC = 12, CD = 3, DA = 6$. E is the intersection of AB and CD, and F is the intersection of AD and BC. Find EF.

4. [5 points] Compute the greatest positive integer n such that n does not end in 0 and the number obtained by erasing the last digit of n is a divisor of n.

5. [6 points] Find an ordered pair of positive integers (x, y) such that $x^7y^8 + x^4y^{14} = 16^7 * 5^8 + 16^4 * 5^{14}$ and $|x| \neq 16, |y| \neq 5$.

6. [7 points] Eight people go out together to see a movie together. At the movie theater, the 8 of them sit in two adjacent rows of 4, so that their group forms a 2×4 rectangle of seats. Because Alex and Albert are best friends, they want to sit adjacent to each other, where adjacent can mean adjacent horizontally or vertically. However, Alex really doesn't like Cole, so they wants to avoid sitting adjacent to each other. In how many different ways can these 8 friends sit?

7. [8 points] Let ΔABC be a triangle with $AB = 5, BC = 7, AC = 8$. Find the radius of the circle that lies in the interior of ΔABC and is tangent to AB, AC, and the incircle of ΔABC.

8. [8 points] What is the remainder when 1248163264128256512 is divided by 1024?

9. [10 points] A round robin tournament has 8 teams, Team 1, Team 2,..., Team 8. If two even numbered teams play then the team with the higher number wins, if two odd numbered teams play the lower number team wins, and if n even numbered team plays against an odd numbered team each team has a $\frac{1}{2}$ probability of winning. A team is called *good* if its team number is equal to its number of wins. What is the maximum number of *good* teams?

10. [10 points] Let ABC be a triangle. Points X and Y on sides AB and AC respectively are chosen such that
$$\frac{BX}{AB} = 2 \cdot \frac{CY}{AC}.$$
A line perpendicular to XY passing through Y intersects BC at P. Given that $\angle PXY = 30^\circ$, find $\angle XPC$.

Team round 2

1. [2 points] A standard 6-sided dice is rolled 10 times. What is the probability that the sum of the numbers rolled is a multiple of 3?
2. [3 points] Let $f(n)$ be the nth prime number. Find $f(f(f(f(2))))$.
3. [5 points] In a game show there are 11 doors, 4 of which hold a prize. After I choose a door, the host opens five other doors, two of which hide a prize. If I choose to switch to another door that's unopened, what's the probability that I pick the door with a prize?
4. [5 points] For a positive integer n, let $\eta(n)$ represent the largest perfect square that divides n. Find
$$\sum_{n|2025} \eta(n)$$
5. [6 points] Compute the least positive integer n such that $\tan 8n^\circ = \dfrac{\cos 35^\circ + \sin 35^\circ}{\cos 35^\circ - \sin 35^\circ}$.
6. [7 points] Two sides of a triangle are 4 and 5, and a midsegment of the triangle is tangent to the incircle. What is the largest possible area of this triangle?
7. [8 points] We have 6 distinct points lying on the diagonals of a square. We then draw all lines connecting each pair of points except for the 2 main diagonals. What is the maximum number of intersections of a pair of lines(including the 6 distinct points)?
8. [8 points] A factor of $z^5 + z + 1$ can be written as $x^3 + ax^2 + bx + c$. Find $a + b + c$.
9. [10 points] Let $ABCD$ be a square with side length 2. Let P be the point outside of $ABCD$ such that $PC = PD$ and $PA : PD = 1 : 2$. Find PA^2.
10. [10 points] Let circle γ be a circle with radius 5 centered at point O, and let circle Γ be a circle with radius 7 also centered at point O. Let point A be on circle γ and points B and C on circle Γ so that $AB = 3$ and $AC = 8$. Find BC^2.