2018 FLSAM PUMaC Tryout

1. Do not look at the test before the round begins.
2. This test consists of 16 short answer problems to be solved in 2 hours. Problems are given in four sets of four, where each set consists of a problem in Algebra, Combinatorics, Geometry, and Number Theory. The problems within each set are of approximately equal difficulty, the sets are ordered in increasing difficulty. There is no penalty for incorrect answers.
3. The problems are weighted by set. Set IV is worth more than Set III, which is worth more than Set II, and so on.
4. Write your name, answers, and other requisite information on the accompanying answer sheet. Also include, if applicable and to the best of your memory, your scores on the 2018 AMC series of tests, making sure to circle whether you took the AMC 10 or AMC 12, and the AIME I or AIME II.
5. No computational aids other than pencil/pen are permitted.
6. Answers are not necessarily integers. All fractional answers should be reduced, and radicals/logarithms should be expressed in simplest possible form.

Answer Sheet

Name: ______________________________

School: ______________________________

Grade: ______________________________

Gender: **M** **F**

2018 AMC (**10** / **12**) A: ____________________

2018 AMC (**10** / **12**) B: ____________________

2018 AIME (**I** / **II**): ____________________

Problem	Answer
A1	
C1	
G1	
N1	

Problem	Answer
A2	
C2	
G2	
N2	

Problem	Answer
A3	
C3	
G3	
N3	

Problem	Answer
A4	
C4	
G4	
N4	

Set I

A1. Mater and John have ages that add up to 31. In 2 years, Maters age will be half as much of what Johns age is then as Maters age was of Johns age 2 years ago. How much older is Mater than John?

C1. How many nonempty subsets of 1, 2, 3, 4, 5, 6, 7 have no pairs of consecutive integers?

G1. Bill the snail is walking around the cartesian plane in a very interesting pattern. He starts at point $(0, 0)$ and walks 1 unit to point $(1, 0)$ then turns 90 degrees to the right, he then walks $\frac{1}{2}$ units before turning 90 degrees to the right again, he continues this pattern of walking half as far each time before turning 90 degrees. What is the ratio of his final distance from the starting point to his total distance traveled?

N1. What is the closest rational to 0.77 that has denominator less than 50?

Set II

A2. What is the greatest prime factor of $82^2 - (18^2 + 2^{12})$?

C2. There are 7 points in the plane, no three of which are collinear. Each pair of points is to be joined by a red or blue line with the following restriction: if A, B, and C are three points and the lines joining AB and AC are both red, then the line joining BC is also red. How many different colorings are possible?

G2. Given triangle ABC with $AB = 13$, $BC = 14$, $CA = 15$, construct a point X such that X is the intersection of the altitude from B and the angle bisector from C. Let Y be the intersection of line AX and CM, where M is the midpoint of AB. Let the centroids of AYB, BYC, and CYA be J, I, and H. Find the area of triangle JIH.

N2. Find the largest integer value of x such that $x^2 + 420x + 1$ is a perfect square.

Set III

A3. Suppose that a, b, c are distinct numbers that satisfy the equation $x^3 - 4x + 8 = 0$. Evaluate $a^5 + b^5 + c^5$.

C3. Harrison the frog is jumping around a number line of lilypads looking for the landmine that just so happens to be on lilypad 20. When Harrison is at lilypad n, where $0 < n < 20$ then every second he stays on lilypad n, moves to lilypad $n + 1$, or moves to lilypad $n - 1$, each with equal probability. When he is at lilypad 0 he either moves to lilypad 1 or stays at lilypad 0, both with equal probability, and when he steps on top of the landmine at lilypad 20 he blows up immediately. Given that Harrison starts at lilypad 3, what is the expected number of seconds it will take for Harrison to blow up?

G3. Let ABC be a triangle with side lengths $AB = 5, AC = 8$, and $BC = 7$. Let E and F be the feet of the B and C altitudes, respectively, and suppose that line EF intersects the circumcircle of ABC at two points P and Q. If I_1 and I_2 are the incenters of $\triangle BPQ$ and $\triangle CPQ$, respectively, find the area of triangle AI_1I_2.

N3. Let $P(x)$ be a monic polynomial with integer coefficients such that $P(n) \equiv 0 \pmod{2025}$ only when $\text{GCD}(2025, n) = 1$. What is the least possible degree of $P(x)$?

Set IV

A4. Let $\{V_n\}$ be a recursive sequence defined by $V_0 = 0$, $V_1 = 2$, and $V_n = 5V_{n-1} - 6V_{n-2} + 2$. Find the value of

$$\sum_{n=0}^{\infty} \frac{V_n}{5^n}$$

C4. Evaluate

$$\binom{50}{7} - \binom{5}{1}\binom{40}{7} + \binom{5}{2}\binom{30}{7} - \binom{5}{3}\binom{20}{7} + \binom{5}{4}\binom{10}{7}$$

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G4. Let $ABCD$ be a cyclic quadrilateral with center O, let $AB \cap CD = P$, let $AD \cap BC = Q$. There exist points X and Y on PQ such that $AX = CX$, and $BY = DY$. Given that $ABCD$ has circumradius 1, $\angle BOC = 35°$, $\angle AOB = 55°$, and $\angle COD = 85°$, what is $XO + YO$?

N4. Let $f(n)$ be the smallest positive integer a that maximizes the expression $\frac{n^a}{(a!)^2}$. Evaluate

$$\sum_{n=1}^{2018} f(n)$$