This test contains a total of 14 short answer questions. Answers will not necessarily be integers, but your answers should be as simplified as possible. Carry out any reasonable calculations, write rational numbers in lowest terms, and simplify radicals.

The questions will be given to you in sets of 2 questions (for a total of 7 sets). You will have 10 minutes to solve each set of questions. The questions are generally arranged in order of difficulty within each set. There is no penalty for giving a wrong answer, so guessing is advantageous.

Do not discuss questions from this test until April 14^{th}. If you choose to do so, you may be giving kids taking a later administration of this test an advantage over you.

Fill in the following information. Write "N/A" if the prompt is not applicable.

Name: ______________________

School: ______________________

Grade: ______________________

Email: ______________________

For this section, please circle one of either "10" or "12" and fill in the following information. Write "N/A" if the prompt is not applicable.

2018 AMC 10 / 12 A Score: ______________________

2018 AMC 10 / 12 B Score: ______________________

2019 AMC 10 / 12 A Score: ______________________

2019 AMC 10 / 12 B Score: ______________________

For this section, please circle one of either "I" or "II" and fill in the following information. Write "N/A" if the prompt is not applicable.

2018 AIME I / II Score: ______________________

2019 AIME I / II Score: ______________________

1. Let $f(x) = x^3 + ax^2 + bx + c$, given that $f(1) = 2, f(2) = 3$, and $f(3) = 1$, what is $a + b + c$?

Answer:____________________

2. A rectangular box has interior dimensions 6 inches by 5 inches by 10 inches. The box is filled with as many solid 3-inch cubes as possible, with all of the cubes entirely inside the rectangular box, and all edges parallel to edges of the box. What percent of the volume of the box is taken up by the cubes?

Answer:____________________

Cont.

3. Harrison has a 19 by 19 grid. How many ways are there for him to color a subset of the 361 squares such that the number of squares colored in each row and column is even?

Answer:______________________________

4. Equilateral triangle ABC is inscribed in a circle. Another triangle DEF is inscribed in the same circle such that angle $\angle DEF = 150^\circ$. ABC and DEF intersect in n points. What is the sum of all possible values of n?

Answer:______________________________

5. Find the number of six-digit integers $\overline{abcdef}$ such that

$$\overline{ab} + \overline{bc} + \overline{cd} + \overline{de} + \overline{ef} = 100$$

Answer:____________________

6. Compute the only positive integer value of $\frac{404}{r^2-4}$, where r is a rational number.

Answer:____________________

7. Compute the least possible value of $m + 100n$, where m and n are positive integers such that

$$\frac{1^2 + 2^2 + \cdots + m^2}{1^2 + 2^2 + \cdots + n^2} = \frac{31}{254}.$$

Answer:____________________

8. Suppose there exist constants A, B, C, and D such that $n^4 = A\binom{n}{4} + B\binom{n}{3} + C\binom{n}{2} + D\binom{n}{1}$ holds true for all positive integers $n \geq 4$. What is $A + B + C + D$?

Answer:____________________

9. A teacher writes the alphabet (in order) on a board, and erases letters in a random order. What is the probability that at some point, i and u will be next to each other?

Answer:______________________________

10. Let a *gang* be a nonempty set G of positive integers with the following properties:

    - There exists an integer c, called the *pump* of G, such that for every integer a in G, there exists integer b in G such that $a + b = c$.
    - All elements of G are pairwise relatively prime.

    A gang G is called *gucci* if there does not exist another gang H with the same pump, such that H is a proper subset of G. Let x be the number of distinct gucci gangs with pump 19 and let y be the number of distinct gucci gangs with pump 21. Find $10y + x$.

Answer:______________________________

11. Bill has a bag with pieces of cake. The bag contains 5 pieces of chocolate cake, 6 pieces of vanilla cake, and 7 pieces of red velvet cake. It takes Bill 1 minute to eat a piece of chocolate cake, 2 minutes to eat a piece of vanilla cake, and 3 minutes to eat a piece of red velvet cake. Given that he eats every piece of cake he picks out, and chooses out of the bag at random, what is the expected amount of time, in minutes, that Bill has been eating cake immediately after he eats his final piece of chocolate cake?

Answer:____________________

12. Let x, y, z be numbers in the range (0,1) such that $xyz = (1-x)(1-y)(1-z)$. Suppose that $\frac{1-x}{x} + \frac{y}{1-y} = 4$ and $\frac{1-y}{y} + \frac{z}{1-z} = 5$. Find $\frac{1-z}{z} + \frac{x}{1-x}$.

Answer:____________________

Cont.

13. In quadrilateral $ABCD$, $AC = BD$ and angle $B = 60°$. Let M and N be the midpoints of AB and CD, respectively. If $MN = 12$ and the area of $ABCD$ is 420, find AC.

Answer:________________________________

14. Consider the hyperbola $x^2 + 4xy + y^2 - 6x - 4 = 0$. What is the 6th smallest possible positive integer value for a such that the line $y = a$ intersects the hyperbola at lattice points?

Answer:________________________________

The End.