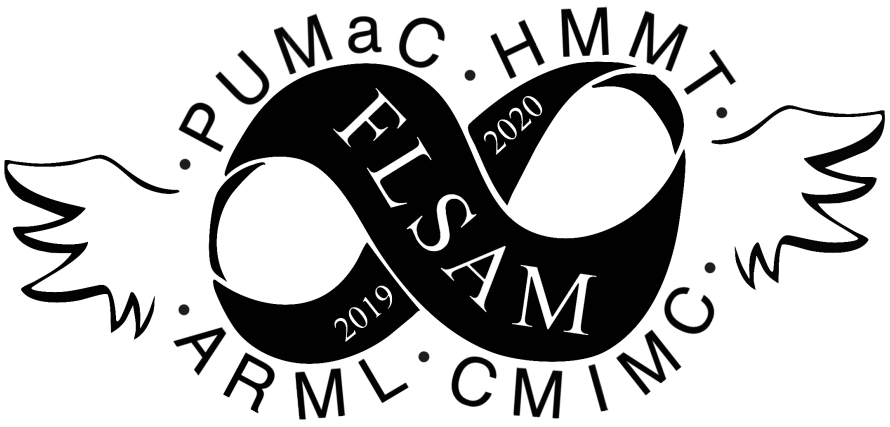
Florida Student Association of Mathematics



2019-2020 Introduction Meeting — August 31, 2019

Team Round

Welcome to the **FLSAM Intro Meeting Team Round!** This event consists of 10 problems that you can work together on. Each problem is worth 1 point.

*You will have **30 minutes** to complete the test. Good luck, and have fun!*

1. A *truncated icosahedron* (the shape associated with a soccer ball) is a three-dimensional solid with 20 hexagonal faces and 12 pentagonal faces. How many edges does a truncated icosahedron have?

2. In a game of Rigged Uno, each card that Yuru gives has a 40% chance of inciting chaos. Saathvik plays a "draw 4" card on Sharvaa, meaning that Yuru gives Sharvaa four cards. What is the probability that at least one of those four cards incites chaos?

3. Points D and E lie on sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle ABC such that $AD = 2BD$ and $CE = 2AE$. Find the measure of $\angle EDB$, in degrees.

4. Let $\underline{SPACE}$ be a 5-digit integer that is the product of the 1-digit number $\underline{E}$ and the 2 digit numbers $\underline{DO}$ and $\underline{IT}$. Find the maximum possible value of $\underline{DESPACITO}$.[1]

5. Yuru, the Greatest Minesweeper Player of All Time$^{\text{TM}}$, is teaching Albert how to play Minesweeper. Albert starts out playing slowly, only being able to clear 100 mines in an hour. The next hour, he improves and is able to clear 200 mines. The hour after that, he clears 300 mines. This pattern of adding 100 mines per hour continues, and Albert clears mines at a constant rate within each hour. If Albert started playing Minesweeper at the moment it turned 12:00, at what time will Albert clear his 2019^{th} mine? Assume the Minesweeper board is very large.

6. In triangle ABC, $AB = 2019$, $BC = 2020$ and $AC = 4000$. Let ω be the circle with diameter $\overline{OA}$ where O is the circumcenter of ABC. The intesections of ω with $\overline{AB}$ and $\overline{AC}$ are C_1 and B_1 respectively. Find B_1C_1.

7. Jae and Iris are having a dice rolling competition. Jae has a fair 7-sided die (with faces numbered 1-7) and Iris has a fair 6-sided die (with faces numbered 1-6). They both roll their dice at the same time. If they both roll the same number, they pick up their dice and roll again. Otherwise, whoever rolled the higher number wins. What is the probability that Jae wins?

8. Let a *gang* be a nonempty set G of positive integers with the following properties:

   - There exists an integer p, called the *pump* of G, such that for any a in G, there exists a b in G such that $a + b = p$.
   - For every a, b in G, $\gcd(a, b) = 1$.

   A gang G is called *gucci* if there does not exist another gang H with the same pump such that G is a proper subset of H. A pump is called *lil'* if it is odd and less than 26. How many gucci gangs have a lil' pump?

9. Let $P(x)$ be a polynomial which satisfies $P(x) = P(0)x^3 + P(1)x^2 + P(2)x + P(0)$. Given that $\frac{P(1)}{P(2)} \neq \frac{7}{8}$, what is the sum of all possible values of $P(2019)$?

10. For any interval $\mathcal{A}$ in the real number line not containing zero, define its *reciprocal* to be the set of numbers of the form $\frac{1}{x}$ where x is an element in $\mathcal{A}$. Compute the number of ordered pairs of positive integers (m, n) with $m < n$ such that the length of the interval $[m, n]$ is 10^{10} times the length of its reciprocal.

[1] All instances of the same letter have the same value.