2018-2019 FLSAM HMMT/CMIMC Tryout

Algebra/Number Theory

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1. Do not look at the test before the round begins.
2. This test consists of 6 short answer problems to be solved in 40 minutes. Problems in this test are from the field of ALGEBRA/NUMBER THEORY.
3. All problems in this test are weighted equally, and ties will be broken by the highest numbered problem solved. There is no penalty for incorrect answers.
4. Write your name, answers, and other requisite information on the accompanying answer sheet. In particular, make sure to circle the appropriate subject of this test. Also include, if applicable and to the best of your memory, your scores on the 2018 AMC series of tests, making sure to circle whether you took the AMC 10 or AMC 12, and the AIME I or AIME II.
5. No computational aids other than pencil/pen are permitted.
6. Answers are not necessarily integers. All fractional answers should be reduced, and radicals/logarithms should be expressed in simplest possible form.

Set 2

1. Define $f(x)$ as sum of the positive integer factors of x and $d(x)$ as sum of digits of x in base 10. How many 2-digit integers n are there such that $f(n)$ and $d(n)$ have the same parity?

2. Let a, b, c, d be pairwise distinct complex numbers such that $|a| = |b| = |c| = |d| = 3$ and $|a + b + c + d| = 8$. Find $|abc + abd + acd + bcd|$.

3. Let $f(x)$ be the least nonnegative integer k such that the sum of the digits of $\frac{x}{2^k}$, including digits after the decimal point, has even remainder when divided by 9. Find $\sum_{n=1}^{2018} f(x)$.

4. Find the remainder when $\prod_{n=1}^{40}(n^2 + n + 1)$ is divided by 41.

5. The sequences $a_0, a_1, a_2...$ and $b_0, b_1, b_2...$ are defined by $a_0 = \alpha$, $b_0 = \beta$, and $a_{n+1} = \alpha a_n - \beta b_n$, and $b_{n+1} = \beta a_{n+1} + \alpha b_n$ for all $n \geq 0$. There are n pairs of real numbers α, β such that $a_{2019} = a_0$ and $b_{2019} = b_0$. Compute the sum of the prime factors of n.

6. Let a, b, c be the roots $x^3 - 3x^2 - 69x - 1 = 0$. Find the value of $(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{b} + \sqrt[3]{c})(\sqrt[3]{c} + \sqrt[3]{a})$.