

2018-2019 FLSAM HMMT/CMIMC Tryout

Combinatorics

1. Do not look at the test before the round begins.
2. This test consists of 6 short answer problems to be solved in 40 minutes. Problems in this test are from the field of COMBINATORICS.
3. All problems in this test are weighted equally, and ties will be broken by the highest numbered problem solved. There is no penalty for incorrect answers.
4. Write your name, answers, and other requisite information on the accompanying answer sheet. In particular, make sure to circle the appropriate subject of this test. Also include, if applicable and to the best of your memory, your scores on the 2018 AMC series of tests, making sure to circle whether you took the AMC 10 or AMC 12, and the AIME I or AIME II.
5. No computational aids other than pencil/pen are permitted.
6. Answers are not necessarily integers. All fractional answers should be reduced, and radicals/logarithms should be expressed in simplest possible form.

Set 3

1. Spamos has a giant cube of side length 2018 that he cuts in half. He takes one of the halves, puts glue all over its outer surface, and cuts it into $\frac{2018^3}{2}$ unit cubes. He then takes one of the unit cubes and throws it on the ground, and if it does not stick to the ground on the first time it lands on a face he picks it up and tries with another cube (if it does, he still tries to pick it up, but finds out that he can't and stops). Find the expected number of throws that he performs before he gets a cube to stick.
2. Chad has 5 balls. He randomly tosses each ball into one of 4 bins. What is the expected number of ball tosses before at least one of the bins contains two balls?
3. Harrison crawls from point A at (0,0) to B at (4, 5) in a sequence of 9 moves, each move being 1 unit right or 1 unit up. He chooses a random one of the possible sequences of moves to make such that each one has equal probability of being chosen. Find the expected number of 90° turns he makes to get from A to B.
4. In a 10×10 grid, there are $\binom{20}{10} = 184756$ paths from the lower left corner to the top right corner. If one of its edges between two adjacent points on the grid is chosen at random and deleted, what is the expected number of paths that remain intact.
5. There are 10 identical crates with dimensions $3 \text{ ft} \times 4 \text{ ft} \times 6 \text{ ft}$. The first crate is placed flat on the floor. Each of the remaining nine crates are placed, in turn, flat on top of the previous crate, and the orientation of each crate is chosen at random. Let $\frac{m}{n}$ be the probability that the stack of crates is exactly 41 ft tall, where m and n are relatively prime positive integers. Find m .
6. Spamos has a strip of paper that is made up of 2048 unit squares in a row from left to right. He folds the paper in half repeatedly with a slap. On each fold, the right edge of the paper is folded over to coincide with the left edge. This is done 11 times. After the last fold, how many squares lie underneath the square that was originally 2018th counting from the left?