2018-2019 FLSAM HMMT/CMIMC Tryout

Geometry

---

1. Do not look at the test before the round begins.
2. This test consists of 6 short answer problems to be solved in 40 minutes. Problems in this test are from the field of GEOMETRY.
3. All problems in this test are weighted equally, and ties will be broken by the highest numbered problem solved. There is no penalty for incorrect answers.
4. Write your name, answers, and other requisite information on the accompanying answer sheet. In particular, make sure to circle the appropriate subject of this test. Also include, if applicable and to the best of your memory, your scores on the 2018 AMC series of tests, making sure to circle whether you took the AMC 10 or AMC 12, and the AIME I or AIME II.
5. No computational aids other than pencil/pen are permitted.
6. Answers are not necessarily integers. All fractional answers should be reduced, and radicals/logarithms should be expressed in simplest possible form.

Set 1

1. In a square with side length 6, what is the distance between the center of the square and the center of the largest equilateral triangle inscribed inside the square?

2. $ABCD$ is a cyclic quadrilateral that has an inscribed circle. The diagonals of $ABCD$ intersect at P. If $AB = 1$, $CD = 4$, and $BP : DP = 3 : 8$, then find the area of the inscribed circle of $ABCD$.

3. There is a right triangle ABC which hypotenuse BC, and a point P inside triangle ABC. Perpendiculars are dropped from P to BC, AC, and AB at points A', B', and C', respectively. Given that $PA' = 3, PB' = 4$, and $PC' = 5$, what is the maximum possible area of $A'B'C'$?

4. Circles ω_1, and ω_2 have radii 3 and 8 respectively, and are externally tangent at point O. What is the length of the locus of the possible centers of circle ω_3 which is tangent to both ω_1 and ω_2, such that ω_3 does not lie on the line between the centers of ω_1 and ω_2?

5. Let MN be the diameter of a circle with radius $\frac{1}{2}$. Points A and B are on the circle on the same side of MN with $MA = \frac{\sqrt{2}}{2}$ and $MB = \frac{8}{17}$. Let point C be on the circle on the other side of MN. AC and BC intersect MN at D and E. Find the largest possible value of DE.

6. Let $\triangle ABC$ have side lengths $AB = 13, AC = 15, BC = 14$. Let D be the touch point of the A-excircle with BC, and suppose that the A-excircle meets with the circumcircle of $\triangle ABC$ at E and F. Lines DE and DF intersect the circumcircle of ABC again at X and Y. Find the area of $\triangle AXY$.