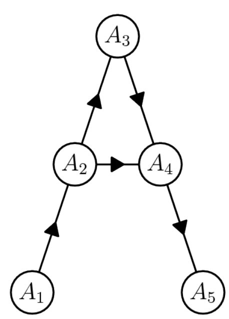
Florida Student Association of Mathematics

2021-2022 Introduction Meeting — September 2021

Ultra Relay

Welcome to the **FLSAM Ultra Relay!** This event consists of 25 problems separated into five different relays. Many of the problems in these relays will depend on other problems, as indicated by the network below. Problems are not all worth the same number of points; the point value for each problem can be found next to the problem.

This round is run like the HMMT Guts Round or the PUMaC Live Round. When the time begins, each team will send a member to pick up copies of the first relay. Once a team has their answers, they send a student to turn in those answers and pick up the next relay. A team **may not** go back to a previous relay after turning in answers, so allocate your time effectively.



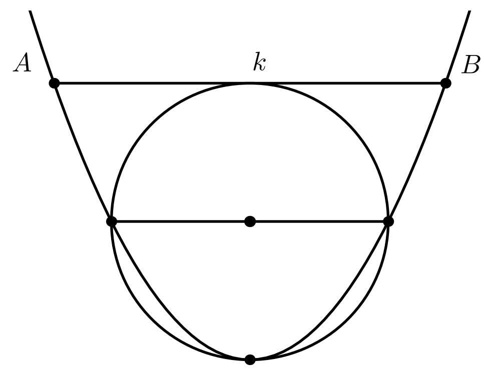
The label for each problem also refers to the answer of that problem. For example, F_1 denotes the answer to problem **F1**.

You will have ***45 minutes*** *to complete the test. Good luck, and have fun!*

A1. **[5]** The number $RICKANDMORTY$ has each distinct letter represent a (not-necessarily distinct) digit in base 16, with $R > 0$. If the sum of all possible values of $RICKANDMORTY$ is x, find the largest integer n such that $2^n \mid x$.

A2. **[7]** Rick decides to send Morty to the school called Amongus Academy. On day 1, Morty needs to travel from point $(0,0)$ to $(\frac{A_1}{3}, \frac{A_1}{3})$. If Morty must visit the point $(6,9)$ and he moves 1 unit in the positive x or y directions at a time, find the number of paths Morty can take modulo 61.

A3. **[9]** A parabola and circle of radius A_2 are shown below, with a segment AB of length k tangent to the circle and having endpoints on the parabola. The indicated diameter of the circle also has endpoints lying on the parabola, and is parallel to AB. Furthermore, the parabola and the circle are tangent at point T. Find $\lfloor k \rfloor$. (That is, find the least integer greater than or equal to k.)



A4. **[10]** Let d be the units digit of A_2. Ritvik takes an isosceles trapezoid $ABCD$ with longer base of length $CD = A_3$ and legs BC, AD of length d. Tushar measures an angle of the trapezoid to be $60°$. He then places two circles of equal radius, with ω_D inscribed in $\angle ADC$ and ω_C inscribed in $\angle BCD$. Ritvik takes another two circles of equal radius to the original two and inscribes ω_A in $\angle DAB$ and ω_B in $\angle CBA$. Now, ω_A is tangent to ω_D at X and ω_B is tangent to ω_C at Y. Then, $XY = \sqrt{a} + \sqrt{b}$ for positive integers a and b. Find $a + b$.

A5. **[12]** Find the sum of all real values of x in the interval $[1, A_4]$ such that $1, \{x\}$, and $\{x^2\}$ form a geometric progression with a non-zero ratio, where $\{x\}$ denotes the fractional part of x.