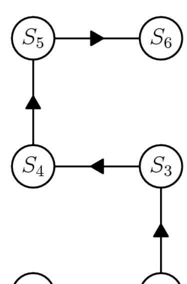
Florida Student Association of Mathematics

2021-2022 Introduction Meeting — September 2021

Ultra Relay

Welcome to the **FLSAM Ultra Relay!** This event consists of 25 problems separated into five different relays. Many of the problems in these relays will depend on other problems, as indicated by the network below. Problems are not all worth the same number of points; the point value for each problem can be found next to the problem.

This round is run like the HMMT Guts Round or the PUMaC Live Round. When the time begins, each team will send a member to pick up copies of the first relay. Once a team has their answers, they send a student to turn in those answers and pick up the next relay. A team **may not** go back to a previous relay after turning in answers, so allocate your time effectively.



The label for each problem also refers to the answer of that problem. For example, F_1 denotes the answer to problem **F1**.

You will have ***45 minutes*** *to complete the test. Good luck, and have fun!*

S1. **[5]** If $a^2 + a + 1 = 0$, evaluate $f(a)$ for $f(x) = x^{37} + 2x^2 + x$.

S2. **[6]** In an Asian household, there are two distinct types of chopsticks, wood and bamboo. Only chopsticks made of different materials are distinguishable. The youngest in the family takes 8 chopsticks uniformly at random. The probability that $|2 \cdot S_1|$ pairs, each pair with two identical chopsticks, can be made is $\frac{4}{17}$. The minimum possible number of total chopsticks is M. If there are M chopsticks, the minimum number of bamboo chopsticks is B and the number of wooden chopsticks is $W = M - B$. Find the product BMW.

S3. **[6]** The distinct prime divisors of S_2 are $p_1, p_2, \ldots, p_k$. Let S be the set of all positive integers of the form $p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$, where all e_i are nonnegative integers. If

$$\sum_{x \in S} \frac{1}{\phi(x)} = \frac{m}{n}$$

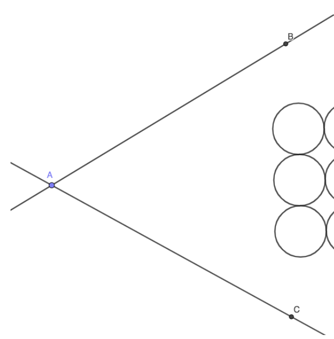
when expressed as a common fraction, find the sum of the (not necessarily distinct) prime divisors of m. (For a positive integer x, $\phi(x)$ is the number of positive integers less than or equal to and relatively prime to x.)

S4. **[8]** Charley gives Aaron the equation $x^3 + S_3x^2 + S_3x + S_3 = 0$. After Aaron solves it, he finds that one of the real roots is within 0.001 to some integer. What is the absolute value of this integer?

S5. **[9]** How many (non-degenerate) triangles with positive integer sidelengths a, b, c satisfy

$$(a^2 + S_4)(b^2 + S_4) = c^2?$$

S6. **[10]** N^2 Survey Corps scouts rush at the Beast Titan with constant speed $5 + S_5$ meters per second. Model each Corps scout as a circle with radius 1 meter, where the centermost circle's center in the front line is initially, at $t = 0$, 501 meters away from the Beast Titan. In their square formation of $N \times N$, where N is an odd positive integer, the circles are tangent to each other.



Every second, with the first throw at $t = 0$, the Beast Titan (modelled as a stationary point) throws rocks in a 60° angle shown above, with the angle symmetric about the horizontal symmetry axis of the formation. Each throw is 1 rock thick, with each rock instantaneously taking out exactly one scout in the front line (given that part of the scout's circle is in the angle) after it is thrown. The back lines are unaffected. The minimum N where at least 1 scout evades all rock throws is n. Find the nearest multiple of 50 to n.