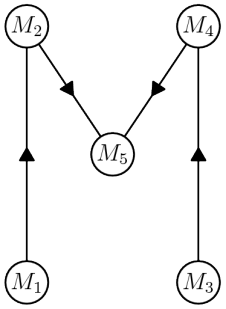
Florida Student Association of Mathematics

2021-2022 Introduction Meeting — September 2021

Ultra Relay

Welcome to the **FLSAM Ultra Relay!** This event consists of 25 problems separated into five different relays. Many of the problems in these relays will depend on other problems, as indicated by the network below. Problems are not all worth the same number of points; the point value for each problem can be found next to the problem.

This round is run like the HMMT Guts Round or the PUMaC Live Round. When the time begins, each team will send a member to pick up copies of the first relay. Once a team has their answers, they send a student to turn in those answers and pick up the next relay. A team **may not** go back to a previous relay after turning in answers, so allocate your time effectively.



The label for each problem also refers to the answer of that problem. For example, F_1 denotes the answer to problem **F1**.

*You will have **45 minutes** to complete the test. Good luck, and have fun!*

M1. [7] Triangle ABC has side lengths $AB = 13, BC = 14$, and $CA = 15$. A circle ω is drawn tangent to both BC at its midpoint and the circumcircle of ABC. If the product of all possible values of the radius of ω is t, find $4t$.

M2. [8] Aaron is pelting a watermelon at the ground. The watermelon has a splash zone of radius M_1. 6 people are standing at the vertices of a hexagon of side length M_1. If Aaron's watermelon lands uniformly at random inside the hexagon and the probability at least 3 people are in the splash zone is $r\sqrt{t}\pi - s$ for rational r and positive integers t and s, with t not divisible by the square of any prime, find $81r + 9t + s$.

M3. [9] Three concentric circles have radii $1, 4$, and 9, with each circle being the orbit of a planet. The period (time for complete revolution around the center) of each orbit satisfies Kepler's third law; the period is proportional to $r^{3/2}$, where r is the radius of the orbit. If the period of the innermost orbit is 1 year, what is the minimal number of years it will take for all planets to be collinear again?

M4. [12] Let $T = \left\lfloor \frac{M_3}{10} \right\rfloor$. Determine the sum of the squares of all real solutions for the following equation, where 10 is the coefficient of $\lfloor x \rfloor$:

$$x^2 - 10\lfloor x \rfloor + \frac{51}{4} = 0.$$

M5. [15] Vismay computes $T = \frac{M_4}{M_2}$. Karthik and Albert are given a polynomial $x^3 + 2\lceil T \rceil x^2 - 20x + 2\lfloor T \rfloor$ and told it has roots pq, qr, rp, where p, q, r are complex numbers. Karthik then finds the monic cubic polynomial with roots p, q, r, while Albert finds the monic cubic polynomial with roots $-p, -q, -r$. They determine that there exists a unique positive real number n such that when put into their polynomials, the outputs are equal. If $n^4 = \frac{a}{b}$ when expressed as a common fraction, find $10a + b$.