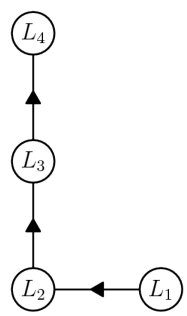
Florida Student Association of Mathematics

2021-2022 Introduction Meeting — September 2021

Ultra Relay

Welcome to the **FLSAM Ultra Relay!** This event consists of 25 problems separated into five different relays. Many of the problems in these relays will depend on other problems, as indicated by the network below. Problems are not all worth the same number of points; the point value for each problem can be found next to the problem.

This round is run like the HMMT Guts Round or the PUMaC Live Round. When the time begins, each team will send a member to pick up copies of the first relay. Once a team has their answers, they send a student to turn in those answers and pick up the next relay. A team **may not** go back to a previous relay after turning in answers, so allocate your time effectively.



The label for each problem also refers to the answer of that problem. For example, F_1 denotes the answer to problem **F1**.

You will have ***45 minutes*** *to complete the test. Good luck, and have fun!*

L1. [**3**] Let $ABCD$ be a square. Extend $\overline{AB}$ past B to a point A' Such that $AA' = 21 \cdot AB$. Similarly, extend $\overline{BC}$ past C, $\overline{CD}$ past D, and $\overline{DA}$ past A to points B', C', and D' respectively such that $BB' = 21 \cdot BC$, $CC' = 21 \cdot CD$, and $DD' = 21 \cdot DA$. Compute the ratio of the area of $A'B'C'D'$ to the area of $ABCD$.

L2. [**6**] Let T be the leftmost digit of L_1. Palm Harbor University High School wants to change their name to be a string with T characters. Each character will be either a, b, or c and the following conditions are met:

- The first character is an a.
- The last character is a c.
- An a and a c never appear consecutively.

Find the number of possible strings.

L3. [**8**] If the rightmost digit of L_2 is d, then let $T = 4d - 1$ be the number of diagonals in a regular polygon. If Peter Pan chooses 2 of these diagonals at random (without replacement) and the probability that they have the same length is $\frac{m}{n}$ when expressed as a common fraction, find $m + n$.

L4. [**10**] Let

$$A = \frac{L_3}{r} + \frac{L_3}{r^2} + \frac{L_3}{r^3} + \cdots, \quad B = \frac{1}{r^2} + \frac{2}{r^4} + \frac{3}{r^6} + \cdots,$$

where r is a real number satisfying $|r| > 1$. Given that $A^2 = B$, the value of $|r|$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Compute $m + n$.