Florida Student Association of Mathematics

2020-2021 Introduction Meeting — August 2x, 2019

Team Round

Welcome to the **FLSAM Intro Meeting Team Round!** This event consists of 10 problems that you can work together on. Each problem is worth 1 point.

*You will have **30 minutes** to complete the test. Good luck, and have fun!*

1. A square piece of cookie dough has length 5. When a circular cookie cutter of radius r is placed such that its center is the same as the center of the dough, pieces of dough are cut off from the corners of the square. Alex takes these pieces and uses them to fill in the spaces between the dough and cutter perfectly (possibly changing the shape of these pieces, but keeping the same area). Assuming the density and depth of the dough is constant, find $r\sqrt{\pi}$.

2. Albert tosses a fair coin and stops when he gets 2 heads in a row. If the probability that Albert stops after exactly 5 tosses is $\frac{m}{n}$ when expressed as a common fraction, find $m+n$.

3. Walter has a rectangular piece of paper $ABCD$. He folds the paper along $\overline{AE}$, where E is a point on $\overline{CD}$ satisfying $EC = 3$, $ED = 5$. Given that D lies on $\overline{BC}$ after the fold, compute AE^2.

4. The *digit sum* of a number can be found by repeatedly taking the sum of digits of the number until a single digit is found. For example, the digit sum of 369 is found as follows: Add the digits to get $3+6+9 = 18$, then add the resulting number's digits to get $1+8 = 9$ as the digit sum. Find the digit sum of 2^{2021}.

5. Pam has made a regular 2022-gon using gumdrops and toothpicks. She chooses 6 distinct vertices. If the expected number of equilateral triangles that has all vertices among these 6 is $\frac{m}{n}$ when expressed as a common fraction, find the sum of the distinct prime factors of mn.

6. In a Mu Alpha Theta club, p people are in the pre-calculus division and c people are in the calculus division. If p has 4 divisors and c has 5 divisors, what is the least value of $p+c$ that has 6 divisors? Here, a divisor is a positive integer divisor.

7. Inside an equilateral triangle ABC, a smaller equilateral triangle XYZ with side length 2 is placed with the opposite orientation such that X is closest to BC, Y is closest to AC, and Z is closest to AB. The distance from X to BC is $\sqrt{3}+1$, from Y to AC is 1, and from Z to AB is 1. If the area of ABC is $a\sqrt{b}+c$ for positive integers a, b, c and b is not divisible by the square of any prime, find $a+b+c$.

8. Page 23 of a notebook has 10 lines. There are 4 distinct names written on the lines, with each line containing exactly 1 name. If each name appears at least once, and the number of ways to fill all 10 lines is N, find $N \pmod{1000}$.

   Since the lines are on a page, they can be distinguished form each other.

9. Find the number of ordered pairs of positive integers (x, y) such that
   $$y(x^2 + 3y^2) = 12x^3,$$
   and $x + y < 2021$.

10. The sum
    $$\sum_{n=1}^{148} \left\lfloor \frac{2^n}{3} \right\rfloor$$
    can be expressed as $\frac{2^a - c}{b}$ for positive integers a, b, c, with c minimized. Compute $a+b+c$.