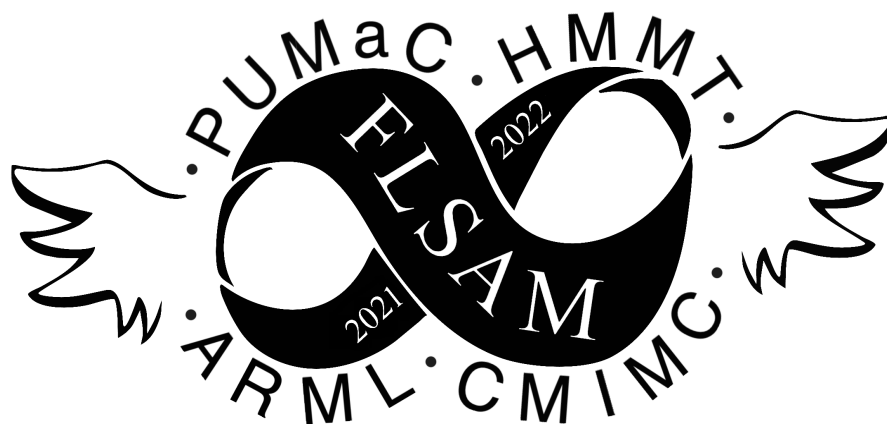


Florida Student Association of Mathematics



2021-2022 Everything Tryout

January 16-23, 2022

Round 1: Algebra and Number Theory

Welcome to the **2021-2022 FLSAM HMMT-PUMaC-CMIMC Tryout!** This is round one of three, each of which will consist of six problems to be completed in 40 minutes. Scoring is simply based on correct answers; there is no penalty for wrong answers. Good luck!

1. Suppose positive integers a, b satisfy $2 \leq a, b \leq 100$. There are exactly 2 positive real solutions of x in the equation

$$\log_{2^a} \log_{2^b} x = \log_{2^{b+1}} \log_{2^{a-1}} x.$$

Find the greatest possible value of ab .

2. Compute the minimum positive integer n which satisfies the following inequality:

$$\frac{(1 + 2 + \cdots + n)(1^3 + 2^3 + \cdots + n^3)}{(1^2 + 2^2 + \cdots + n^2)^2} > 1.124$$

3. For a positive integer n , consider variables a_1, a_2, \dots, a_n , and the expression $(\pm a_1 \pm a_2 \pm \cdots \pm a_n)^2$, where each \pm sign has an equal probability of representing a plus or a minus. Let $f(n)$ denote the expected value of the number of terms of the expanded expression (with all like terms combined) with a positive coefficient. Compute the least positive integer n such that $f(n)$ is an integer, and it is a multiple of 20.
4. Find the largest positive integer n such that there exist distinct positive integers a_1, a_2, \dots, a_n satisfying

$$a_1 a_2 \cdots a_n = 2021^{2021}.$$

5. Suppose that a, b are positive integers such that $\sqrt{2^{76} - 2^{51} - 2^{50} + 1} = a\sqrt{b}$, and b is not divisible by the square of any prime. b is the product of a 1-digit prime, a 2-digit prime, a 3-digit prime, and a 4-digit prime. Compute this 4-digit prime number.
6. For a positive integer $k \geq 2$, let a_k, b_k and c_k be the complex roots of the equation below.

$$\left(x - \frac{1}{k-1}\right) \left(x - \frac{1}{k}\right) \left(x - \frac{1}{k+1}\right) = \frac{1}{k}$$

Given that the sum

$$\sum_{k=2}^{\infty} \frac{(a_k^2 + a_k)(b_k^2 + b_k)(c_k^2 + c_k)}{k+1}$$

can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, determine $m+n$.