# Florida Student Association of Mathematics

## 2021-2022 Everything Tryout

January 16-23, 2022

*Round 1: Algebra and Number Theory*

Welcome to the **2021-2022 FLSAM HMMT-PUMaC-CMIMC Tryout!** This is round one of three, each of which will consist of six problems to be completed in 40 minutes. Scoring is simply based on correct answers; there is no penalty for wrong answers. Good luck!

**1.** Suppose positive integers $a, b$ satisfy $2 \leq a, b \leq 100$. There are exactly 2 positive real solutions of $x$ in the equation

$$\log_{2^a} \log_{2^b} x = \log_{2^{b+1}} \log_{2^{a-1}} x.$$

Find the greatest possible value of $ab$.

**2.** Compute the minimum positive integer $n$ which satisfies the following inequality:

$$\frac{(1+2+\cdots+n)(1^3+2^3+\cdots+n^3)}{(1^2+2^2+\cdots+n^2)^2} > 1.124$$

**3.** For a positive integer $n$, consider variables $a_1, a_2, \cdots a_n$, and the expression $(\pm a_1 \pm a_2 \pm \cdots \pm a_n)^2$, where each $\pm$ sign has an equal probability of representing a plus or a minus. Let $f(n)$ denote the expected value of the number of terms of the expanded expression (with all like terms combined) with a positive coefficient. Compute the least positive integer $n$ such that $f(n)$ is an integer, and it is a multiple of 20.

**4.** Find the largest positive integer $n$ such that there exist distinct positive integers $a_1, a_2, \ldots, a_n$ satisfying

$$a_1 a_2 \cdots a_n = 2021^{2021}.$$

**5.** Suppose that $a, b$ are positive integers such that $\sqrt{2^{76} - 2^{51} - 2^{50} + 1} = a\sqrt{b}$, and $b$ is not divisible by the square of any prime. $b$ is the product of a 1-digit prime, a 2-digit prime, a 3-digit prime, and a 4-digit prime. Compute this 4-digit prime number.

**6.** For a positive integer $k \geq 2$, let $a_k, b_k$ and $c_k$ be the complex roots of the equation below.

$$\left(x - \frac{1}{k-1}\right)\left(x - \frac{1}{k}\right)\left(x - \frac{1}{k+1}\right) = \frac{1}{k}$$

Given that the sum

$$\sum_{k=2}^{\infty} \frac{(a_k^2 + a_k)(b_k^2 + b_k)(c_k^2 + c_k)}{k+1}$$

can be written as $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers, determine $m + n$.