Florida Student Association of Mathematics

2021-2022 Everything Tryout

January 16-23, 2022

Round 2: Geometry

1. Triangle ABC, with side lengths $AB = 5, BC = 8$, and $CA = 7$ is circumscribed by circle ω. Another point A' on ω is chosen such that ABC and $A'BC$ have the same area. Find AA'.

2. Consider an ellipse $\mathcal{E}$ with a horizontal major axis of length $2a$, where a is a positive real number. A circle Ω is inscribed tangent to the the endpoints of the vertical minor axis. Another circle, ω, of radius r, is entirely contained in $\mathcal{E}$, externally tangent to Ω and internally tangent to $\mathcal{E}$ at one endpoint of the major axis. Find the minimum possible value of the ratio a/r.

3. Consider $\triangle ABC$ with $AB = AC = 20\sqrt{21}$ and $BC = 40\sqrt{7}$. Let D, E denote the projections of A, B onto $\overline{BC}, \overline{CA}$, respectively. Let lines DE and AB intersect at P. Given that PD can be expressed as $a\sqrt{b}$ for positive integers a, b with square-free b, compute $a + b$.

4. In acute triangle $\triangle ABC$, let H be its orthocenter (intersection of altitudes), and let O be its circumcenter (intersection of perpendicular bisectors of sides). Circle (AHO) is tangent to (ABC), $\tan A = \frac{26}{15}$, and $BC = 52$. Compute the area of $\triangle ABC$.

5. Consider $\triangle ABC$ with $AB = 13$, $BC = 14$, $CA = 15$. Let M denote the midpoint of $\overline{AC}$, and let E be an variable point on line $\overline{AB}$ such that $AE < EC$. Let D denote the point on $\overline{BC}$ such that $\overline{AD}$, $\overline{BM}$, $\overline{CE}$ concur and let F denote the point on $\overline{AC}$ such that $\overline{DF} \parallel \overline{AB}$. The maximum possible area of $\triangle EFM$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $100m + n$.

6. Convex cyclic quadrilateral $APQB$ with vertices in that order is inscribed in circle ω with center O. There is a point T on segment $\overline{AB}$ such that $AT = AP$, $BT = BQ$, and the midpoint M of $\overline{PQ}$ satisfies $MT \perp AB$. Given $MO = 41$ and $MT = 40$, compute the sum of all possible values of TO.