Florida Student Association of Mathematics

2022 ARML Tryout

April 2022

Welcome to the **2022 FLSAM ARML Tryout!** The tryout will consist of **7 sets** of **2 problems each**. You will have **10 minutes** to work on each set. Write your name and answers directly on each problem set. Scoring is based on the number of correct answers; there is no penalty for wrong answers. Good luck!

Round 1

Name: ______________________

1. ______________ **2.** ______________

1. Alex is reading the book *1984*, which has 328 pages, which are numbered $1 - 328$. He reads a consecutive set of pages such that the sum of the page numbers on the pages he reads equals 1984. Find the page number he started on.

2. In a circle, there exist 4 distinct chords of length 8 such that the chords enclose a square with area 36. The area of the circle can be expressed as $a\pi$, where a is a positive integer. Compute a.

Round 2

Name: ______________________

3. ____________ **4.** ____________

3. Let m and n be distinct positive integers. The sum of the positive integer divisors of m is equal to the sum of the positive integer divisors of n. Find the least possible value of $m + n$.

4. In right triangle ABC, we have $\angle B = 90^\circ$ and $AB = BC = 4$. Let D be a point on AC such that $BD = 3$. Compute $AD^2 + CD^2$.

Round 3

Name: ______________________

5. ______________ **6.** ______________

5. Compute the sum of the distinct prime factors of 163681.

6. Consider the polynomial $x^5 + x^4 + x^3 + x^2 + x + 1$. Suppose $P_n = a^n + b^n + c^n + d^n + e^n$, where a, b, c, d, e are the roots of the polynomial. Find $P_5 + P_4 + P_2 + P_1$.

Round 4

Name: ______________________

7. ______________ **8.** ______________

7. Let the nth triangular number $T_n = 1 + 2 + 3 + \cdots + n$ and

$$P_n = \frac{T_2}{T_2 - 1} \cdot \frac{T_3}{T_3 - 1} \cdot \cdots \cdot \frac{T_n}{T_n - 1}.$$

Given that the product P_{2022} can be written as $\frac{m}{n}$ for relatively prime positive integers m and n, determine $m + n$.

8. Find the sum of all positive integers $x < 43$ such that $2x^3 + 2x + 1$ is divisible by 43.

Round 5

Name: ____________________

9. ______________ **10.** ______________

9. Consider the set $S = \{1, 2, 3, \ldots, 20\}$. Adam selects 3 distinct elements a, b, c from S. Let $d = \gcd(a, b, c)$. If the probability that d is prime is $\frac{m}{n}$ for relatively prime positive integers m and n, find $100m + n$.

10. Three points A, B, C are marked on a sphere Ψ with diameter at least π is such that the shortest distance between A, B traveling on the surface of Ψ is 2, the shortest distance between B, C is 3, and the distance between C, A is 4. In space $\angle ABC = 90^\circ$; that is, the angle between line segments $\overline{AB}, \overline{BC}$ is 90°. What is the ratio of the surface area to the radius of Ψ?

Round 6

Name: ______________________

11. ______________ **12.** ______________

11. Suppose that x and y are real numbers which satisfy the following

$$4^x + (\sqrt{2})^x = 84$$

$$9^y + (\sqrt{3})^y = 260$$

Compute xy.

12. Consider the following operation I on a positive integer n: for the largest integer k satisfying $2^k \mid n$, replace n with $n + 2^k$. For instance, $I(5) = 6$ and $I(6) = 8$. Let $f(n)$ be the least number of applications of I to n necessary to turn it into a power of 2. For instance, $f(1) = f(2) = f(4) = 0$ since $1, 2, 4$ are already powers of 2, and $f(5) = 2$. Compute $\sum_{n=1}^{512} f(n)$.

Round 7

Name: ____________________

13. ______________ **14.** ______________

13. With the sun directly overhead, a cube suspended in the air casts a shadow on flat ground, forming a hexagon with side lengths 4,1,4,4,1,4 in clockwise order. Compute the surface area of the cube.

14. Let $f(x)$ denote the 3 leading digits of x for integer $x \geq 100$. For instance, $f(420) = 420$ and $f(1337) = 133$. How many possible values can the ordered pair $(f(2^k), f(5^k))$ take for positive integer $k \geq 3^{3^3}$?