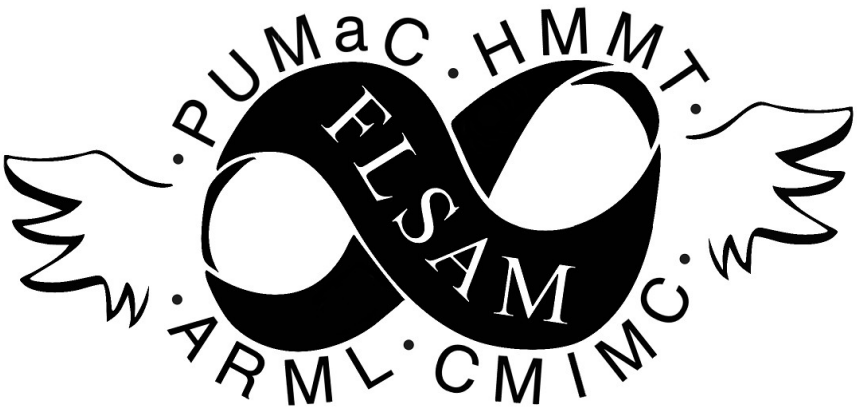
Florida Student Association of Mathematics



2024 ARML Tryout

March 2024

Welcome to the **2024 FLSAM ARML Tryout!** The tryout will consist of **7 sets** of **2 problems each**. You will have **10 minutes** to work on each set. Write your name and answers directly on each problem set. Scoring is based on the number of correct answers; there is no penalty for wrong answers. Good luck!

Round 1

Name: _____

1. _____ **2.** _____

1. How many distinguishable permutations of *BONOBO* are there such that no two adjacent letters are the same?

2. Consider the incenter I, of equilateral triangle ABC with side length 7. A point D inside the triangle satisfies $AD = 5$ and $BD = 3$. Given that $DI = \frac{m}{\sqrt{n}}$ when written in simplest form, find $m + n$.

Round 2

Name: _____

3. _____ **4.** _____

3. Let $ABCD$ be a square and X and Z be points on sides $\overline{BC}$ and $\overline{AD}$ respectively. There exists a point Y inside the square such that C lies on line YZ and quadrilateral $AXYZ$ is an isosceles trapezoid. If $AB = 12$ and $CX = 7$, and $YZ = \frac{m}{n}$ in simplest terms, what is $m + n$?

4. The graphs of $f(x) = a|x|$ and $g(x) = -b|x| + 22$ are drawn, where a and b are positive reals. Given that the finite area of the region bounded by $f(x)$ and $g(x)$ is 2024, and $a + b = \frac{m}{n}$ when expressed in simplest form, find $m + n$.

Round 3

Name: _____

5. _____ **6.** _____

5. Distinct digits A, F, L, M, R, S, T satisfy $\overline{ARML} - \overline{SMT} = \overline{AFT}$. What is the maximum value of the five digit number $\overline{FLSAM}$?

6. Suppose k is a complex number such that the function $f(x) = x^3 + kx^2 + 20x + 24$ has one root that is the average of the other two. Compute the magnitude of the product of all possible values of k.

Round 4

Name: _____

7. _____ **8.** _____

7. Find the sum of all n such that $n! + 9$ is a perfect cube.

8. Two parabolas $f_1(x) = a_1x^2 + b_1x + c_1$ and $f_2(x) = a_2x^2 + b_2x + c_2$ have each of their coefficients chosen (with replacement) from the set $\{1, 2, 3, 4, 5, 6\}$. For how many ordered pairs $(f_1(x), f_2(x))$ do $f_1(x)$ and $f_2(x)$ have exactly one point in common?

Round 5

Name: _____

9. _____ **10.** _____

9. Let $ABCD$ be a rectangle with M the midpoint of $\overline{CD}$. Suppose that the angle bisectors of $\angle MAB$ and $\angle CMA$ intersect at the midpoint of $\overline{BC}$. If $AM = 24$, what is BD^2?

10. Consider the polynomial $f(x) = x^3 + x^2 + 1$, with roots a, b, and c. Given that

$$M = \frac{a}{b^3+b} + \frac{b}{c^3+c} + \frac{c}{a^3+a} + \frac{a}{c^3+c} + \frac{b}{a^3+a} + \frac{c}{b^3+b},$$

find $\lfloor 100M^2 \rfloor$.

Round 6

Name: _____

11. _____ **12.** _____

11. Compute the number of nonempty subsets of $\{1, 2, \ldots, 10\}$ for which the sum of its smallest element and largest element exceeds 10.

12. Let z_1, z_2, z_3 be complex numbers each with magnitude 1 satisfying

$$\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1} = 1.$$

Find the largest possible value of $|9z_1 + 15z_2 + 25z_3|$.

Round 7

Name: _____

13. _____ **14.** _____

13. Triangle ABC has side lengths $AB = 26$, $BC = 28$, and $CA = 30$. Let ℓ be the angle bisector of $\angle BAC$, and let D, E denote the projections of B, C onto ℓ, respectively. The circumcircles $\triangle BCD$ and $\triangle BCE$ intersect ℓ a second time at F, G, respectively. Given that $FG = a\sqrt{b}$ in simplest form, find $a + b$.

14. Tiger writes the numbers $1, 2, \ldots, 2024$ in order on a board. In a *switcheroo*, he may swap the positions of any two numbers on the board. He performs n switcheroos, and then takes the function $f\colon \{1, \ldots, 2024\} \longrightarrow \{1, \ldots, 2024\}$ such that the ith number written on the board is $f(i)$. It turns out that for any integers $1 \leq i \leq 2024$ and $1 \leq k \leq 100$, $f^k(i) \neq i$. Find the smallest possible value of n.