Florida Student Association of Mathematics

2023-2024 HMMT/CMIMC Tryout

December 16-17, 2023

Round 1: Algebra

Welcome to the **FLSAM HMMT/CMIMC Tryout!** The tryout has 3 rounds, each of which will consist of 6 problems to be completed in 40 minutes. Scoring is based on correct answers; there is no penalty for wrong answers. All answers are positive integers. Good luck!

1. If the value of x satisfying

$$2(5(2(5(2(5x-4)+3)-4)+3)-4)+3=x$$

can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n, compute $m+n$.

2. Compute the sum of all reals a such that there exists a quadratic P with leading coefficient 1 satisfying $P(a)=10$, $P(10)=100$, $P(100)=a$.

3. Find the smallest positive integer x such that $\binom{5043}{7}+x$ is a perfect square.

4. On the graph of $y=x^4-10x^3+15x^2+10x-6$, two distinct points $P=(1,10)$ and Q, both with integer coordinates, are marked. If the distance PQ is also an integer, find the sum of all possible x-coordinates of Q.

5. If $425^2+312^2=520^2+87^2$ can be written as the product of two distinct primes, what is their sum?

6. The smallest positive solution of the equation below can be written as $a\pi/b$, with a, b relatively prime positive integers. Find $10a+b$.

$$\tan\left(\frac{\pi}{6}+x\right)\tan\left(\frac{\pi}{6}+3x\right)\tan\left(\frac{\pi}{6}+9x\right)=\tan\left(\frac{\pi}{3}+x\right)\tan\left(\frac{\pi}{3}+3x\right)\tan\left(\frac{\pi}{3}+9x\right).$$