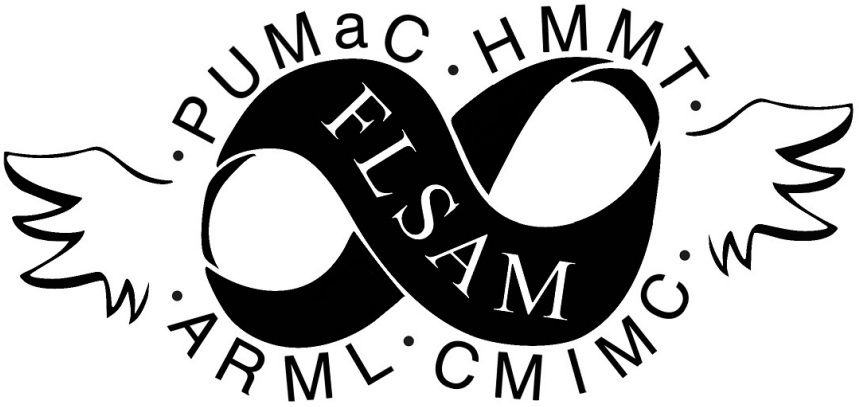
Florida Student Association of Mathematics



2023-2024 HMMT/CMIMC Tryout

December 16-17, 2023

Round 3: Combinatorics

1. Tiger has a coin that has an unknown probability of flipping heads. Andrew then tells Tiger that the probability of flipping heads is a random fraction in the infinite set $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$. Given that Tiger gets three heads after flipping the coin three times and the probability of the coin being fair is $p = \frac{m}{n}$ for relatively prime positive integers m and n, find $10m + n$.

2. In $\triangle ABC$, we have $AB = BC = 1$ and $AC = \sqrt{2} \approx 1.4142$. Points P and Q are chosen uniformly on $\overline{AB}$ and $\overline{BC}$, respectively. Let p be the probability that the perimeter of $ACQP$ is less than BPQ. Compute $\lfloor 1000p \rfloor$.

3. Alex is practicing basketball for 100 days. He makes a layup every x days, starting on Day 1, and makes a 3-pointer every y days, starting on Day 2. How many ordered pairs of integers (x, y) with $1 \leq x, y \leq 10$ are there such that there will be a day where he makes a layup and a 3-pointer?

4. Compute the last three digits of the number of tuples of integers $(a_1, a_2, \ldots, a_6)$ such that $0 \leq a_i \leq 11$ and $a_1 + 2a_2 + \cdots + 6a_6$ is divisible by 7.

5. A bird wants to walk from $(0, 0)$ to $(12, 12)$ using only unit steps going right and up. Suitable paths do not pass through any point with odd x and y coordinates that sum to 12. How many more suitable paths are there than unsuitable paths?

6. Determine the number of ways to permute AAaaBBbb so that:

- the number of A/a's between the two B's is even,
- the number of A/a's between the two b's is odd,
- the number of B/b's between the two A's is even,
- the number of B/b's between the two a's is odd.

For example, abBBabAA is a valid permutation.