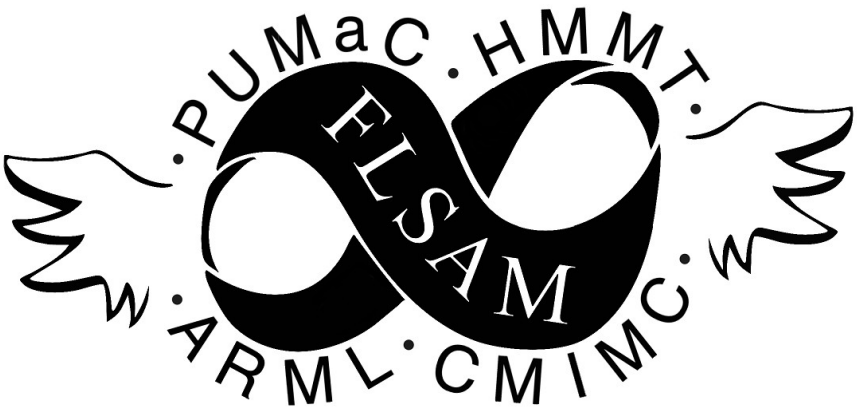
Florida Student Association of Mathematics



2023-2024 Introduction Meeting — September 2023

Team Round

Welcome to the **FLSAM Intro Meeting Team Round!** This event consists of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point.

*You will have **60 minutes** to complete the test. Good luck, and have fun!*

1. There is a world where people are either truth tellers who always tell the truth, liars who always lie, or alternators who change the truth value of their responses every time they respond. A group of people were each asked if they were a truth teller, and then asked if they were an alternator. If 75 people said yes to being a truth teller and 61 people said yes to being an alternator, how many truth tellers were in the group?

2. Aaron stands blindfolded near the edge of a cliff so that if he takes one step right he will fall off. Because he likes living on the edge, every minute he moves to the right one step with probability $\frac{3}{4}$ and moves to the left two steps with probability $\frac{1}{4}$. What is the expected number of minutes Aaron needs to fall off the cliff?

3. Consider $\triangle ABC$ with $AB = 13$, $BC = 14$, $CA = 15$. Let M be the midpoint of the arc BC containing A in the circumcircle of $\triangle ABC$. If X is the unique point on segment AC with $MA = MX$, find CX^2.

4. Find the sum of all reals x such that $\sqrt[3]{2024 - x} + \sqrt{x - 2023} = 1$.

5. A magical ant is located at the origin. Every second, it can teleport a positive integer distance in either the positive x, y, or z direction, but cannot teleport in the same direction twice in a row. Find the number of ways the ant can get to $(2, 3, 5)$.

6. Let $P(x)$ be a monic quadratic polynomial with positive integer coefficients satisfying

$$n + 2 \mid P(n) + P(n + 1)$$

for all positive integers n. Compute the minimum possible value of $P(10)$.

7. Suppose Maximillian the monkey has an infinite supply of bananas. Every 25 seconds, Maximillian eats a banana either in one or two bites. However, if he ever eats three bananas in a row in three bites, Maximillian will throw up and falls asleep. Let $\mathbb{E}$ be the expected number of total bites Maximillian takes before he throws up. If the probability Maximillian eats a banana in one bite is $\frac{3}{5}$, $\mathbb{E}$ can be written in simplest form as $\frac{m}{n}$. Find $m + n$.

8. A country has 101 cities, each pair connected by a two-way flight by either Company A or B. If 30 of the flights from each city are supplied by Company A, how many triples of cities are there such that among the 3 direct flights between them, there are flights by both companies?

9. Isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$ has an inscribed circle ω with center I. Given $BC = 50$ and $IB = 30$, find the radius of ω.

10. Each square's side length is exactly one-quarter of the one adjacent to it from the previous iteration. If the side length of the base square is 1, what is the total perimeter of the figure as the pattern goes to infinity?

