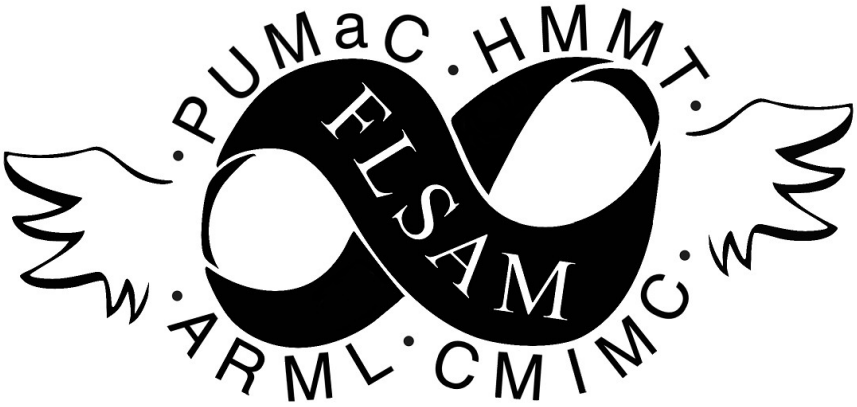
Florida Student Association of Mathematics



2023-2024 PUMaC Tryout

October 14-15, 2023

Round 2: Combinatorics

1. Tiger has four $4-$sided dice, each with faces labeled $1-4$, and Andrew has four $6-$sided dice, each with faces labeled $1-6$. They play a game where they both roll all their dice, and whoever has the (single) die with the highest outcome wins. If there's a tie, they flip a fair coin to determine the winner. If the probability that Andrew wins can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n, what is $m+n$?

2. How many five-digit positive integers exist such that its nonzero digits are strictly decreasing from left to right?

3. Some cells of a 71×71 grid of squares are colored blue so that any blue cell shares at least one vertex with at most one other blue cell. Find the greatest possible number of blue cells.

4. A row of 2023 buckets are all empty except for the middle bucket. In one move, one can choose any bucket except the first and last and transfer $\frac{2}{3}$ of its contents to its left neighbor and $\frac{1}{3}$ of its contents to its right neighbor. If N is the minimum integer number of gallons of water that one must start with to move at least 2023 gallons of water into the rightmost bucket, find the last three digits of N.