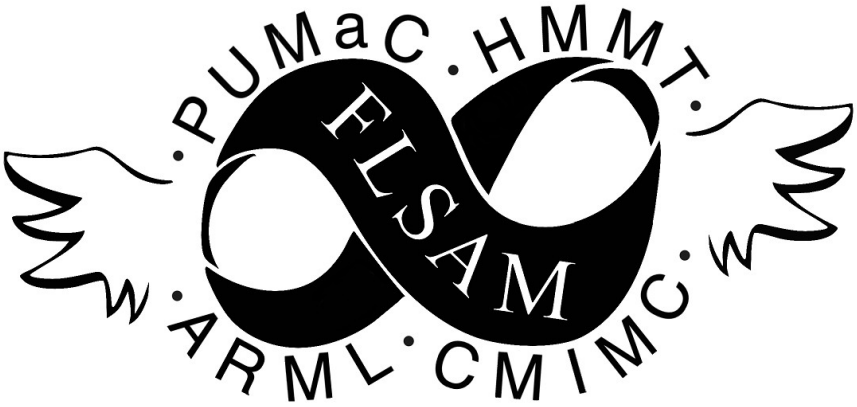
Florida Student Association of Mathematics



2023-2024 PUMaC Tryout

October 14-15, 2023

Round 3: Geometry

1. Let $ABCD$ be an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$ and circumcenter O. Given that $\angle AOB = 42^\circ$ and $\angle COD = 138^\circ$, what is minimum possible value of $\angle ABC$ in degrees?

2. Let $ABCD$ be a parallelogram with $AB = 5$, $BC = 8$, and $CA = 7$. Let O denote the circumcenter of $\triangle ABC$. If OD^2 can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n, what is $m + n$?

3. Two clocks, C_1, C_2, with radii 2023 touch at point P at points 3 and 9 o'clock on the circles, respectively. An ant at P walks counterclockwise along C_1 at the same pace as another ant walking clockwise starting from the 1 o'clock position on C_2. As they walk indefinitely, the midpoint of their positions traces a finite locus. Find the length of this locus.

4. Consider acute $\triangle ABC$ with orthocenter H. Suppose $\odot(BHC)$ intersects AB, AC a second time at D, E, respectively. If $AH = 13$ and $DE = 24$, then BC can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Compute $m + n$.