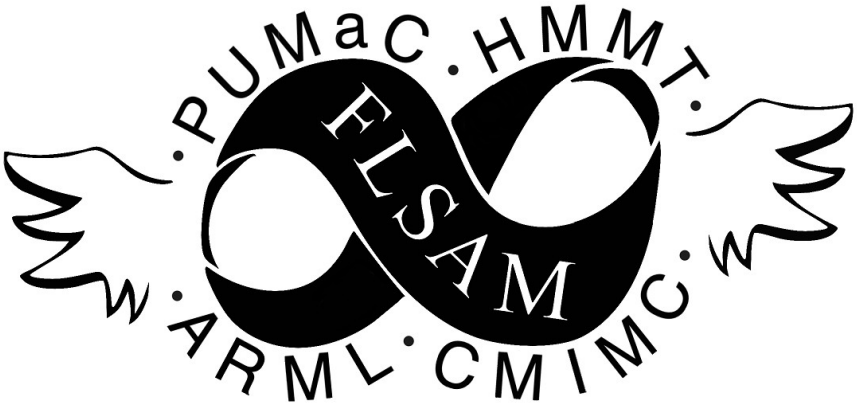
Florida Student Association of Mathematics



2023-2024 PUMaC Tryout

October 14-15, 2023

Round 4: Number Theory

1. Consider the $2n$-digit integer $K = 202323\ldots23$, where the digits repeat with 23s after the initial 20. Find the least positive integer n such that K is divisible by 99.

2. How many integers $1 < n \leq 10000$ are such that $p^2 + 2 \mid n$, where p is the smallest prime factor of n?

3. Compute the sum of the distinct prime factors of $12^8 - 1$.

4. Sequences a_n, b_n with $a_1 = b_1 = 1$ satisfy the recurrences $a_{n+1} = a_n + 2b_n$ and $b_{n+1} = a_n + b_n$. Find the remainder when $a_{306} \cdot b_{306}$ is divided by 17.