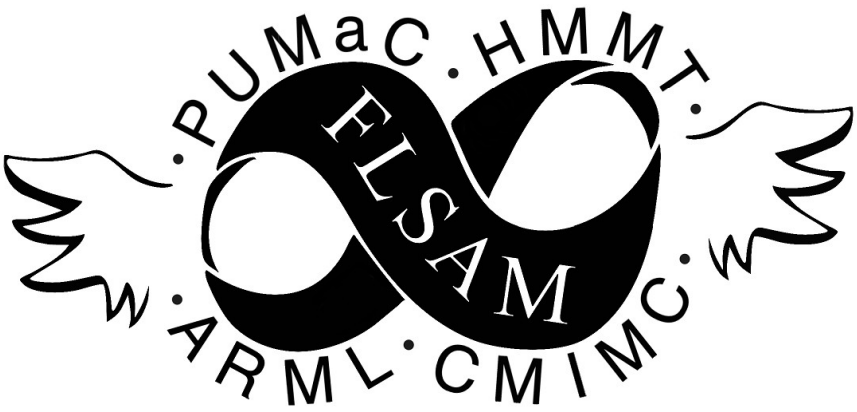
Florida Student Association of Mathematics



2023 Spooky Halloween Contest Individual Round

October 2023

Welcome to the **2023 FLSAM Spooky Halloween Contest!** This is the individual round, consisting of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty.

*You will have **40 minutes** to complete the test. Good luck, and have fun!*

1. A sorcerer can allocate 15 mana between the range, r, and strength, s, of a spell. If the overall effect is $rs + 2r + 3s$, find the maximum possible effect.

2. Andrew is addicted to eating candy corn. His left hand can hold 11 candy corn, his right hand can hold 13 candy corn. Every minute, he eats all the candy corn one of his hands can hold, alternating between his left and right hands and starting with his left. What is the smallest perfect square amount of candy corn he could have eaten after any positive integer number of minutes?

3. Yuhan is trick-or-treating. Each house has 50% chance of tricking him and 50% chance of giving him a treat. When he is tricked, he loses all the treats he has gathered thus far and begins a new streak. If the probability Yuhan gathers four treats in one streak before being tricked three times is $\frac{p}{q}$ for relatively prime positive integers p, q, what is $p + q$?

4. A blueprint for a haunted house is arranged in a 2×2 grid. In each room a silly architect writes a positive integer number of spooky, scary skeletons. On both sides of each row, and on the top and bottom of each column, he writes the product of the two numbers in the row or column. The sum of all twelve written numbers is 2023. What is the smallest possible number of skeletons in the grid?

5. In triangle AEI, points V, L are constructed on line EI such that $IV = IL = EI/2$, arranged in order E, V, I, L. Further, $S \neq A$ is constructed on AE with $ES = EA$. If $AV = EI = 8$ and $AE = 9$, find SL^2.

6. A ghost has a string "OBOBO...OBOBO" which includes 17 O's and 16 B's. Having discovered a love for the palindromic, find the number of ways it can delete all but 3 characters of the string to leave the word "OBO".

7. Halt! A sphinx demands you answer the following riddle: A monic quadratic polynomial $P(x)$ has two distinct integer roots, whereas $P(P(x))$ only has one root. If $P(0)$ is a multiple of 2024, what is the smallest possible value of $\frac{P(0)}{2024}$?

8. Jayden's favorite non-zero integers a, b, and c satisfy the spooky property that

$$\frac{a+b-c}{c} = \frac{b+c-a}{a} = \frac{a+c-b}{b}.$$

Find the sum of all the possible values of $\frac{(a+b)(b+c)(c+a)}{abc}$.

9. Let ABC be an equilateral triangle with side length 2023. Points D, E lie on ray AC and F, G lie on ray AB such that $DC = CE = FB = BG$, $AD < AE$, and $AF < AG$. The center of the circumcircle of DAG lies on the circumcircle of EAF. Given that the length of CD can be written as $\frac{m}{n}$ in simplest form, find $m + n$.

10. A dragon has stolen most of the combo problems from this test. It left behind a clue that reads: $y^2 + 3x = 9x^3y^4$ for real numbers x and y. If the minimum value of $\frac{1}{2x} + \frac{3}{y^2}$ is M, find M^4 for any hope of the subject reappearing on FLOMO.