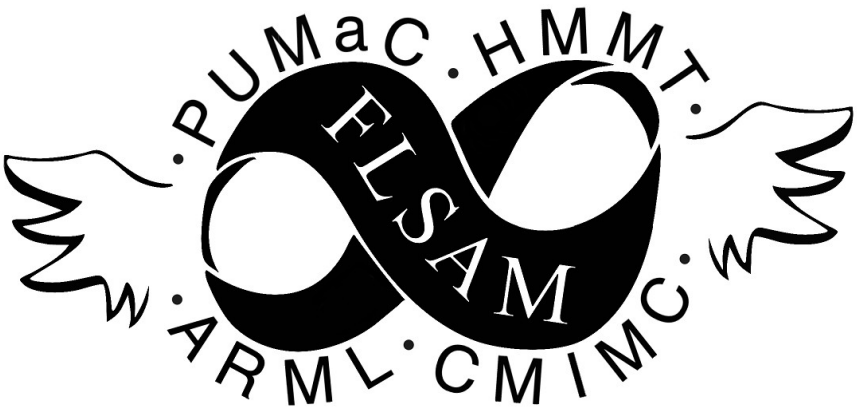
Florida Student Association of Mathematics



2023 Spooky Halloween Contest Relay

October 2023

This is the relay, consisting of two sets of three problems. For each set, the three members of a team will receive different problems, with the second and third problems relying on the answer of the previous problem to solve. You may work on your problem before receiving an answer from your teammate.

*Your team will have **5 minutes** to complete each set. Good luck, and have fun!*

Relay Set 1

1. In Transylvania, vampire coffins are given 2-digit numbers with digits in non-decreasing order when read left to right. If no two coffins can have the same number, how many coffins can be built?

2. Let 🙂 be the number you will receive. How many positive integers $n \leq 900$ satisfy:

$$\left\lfloor \frac{n🙂}{11} \right\rfloor = \left\lfloor \frac{(n-1)🙂}{11} \right\rfloor + 5\,?$$

3. Let 🙂 be the number you will receive. If x, y are rational solutions to $xy(xy+4) - 7(x^2+y^2) + 🙂 = 0$, find $|x| + |y|$.

Relay Set 2

4. Define $\overset{x,y,z}{\text{🎃}} = y + z - 2x$. If b, a, t are the roots of $x^3 - 4x^2 + x + 6$, evaluate

$$\overset{b,a,t}{\text{🎃}} \times \overset{a,t,b}{\text{🎃}} \times \overset{t,b,a}{\text{🎃}}.$$

5. Let 💀 be the number you will receive. Consider the infinite sequence $\frac{10💀}{1}, \frac{10💀}{4}, \frac{10💀}{7}, \cdots$ consisting of $10 \cdot 💀$ divided by elements of an arithmetic sequence. Find the number of integer elements of the sequence.

6. Let 💀 be the number you will receive. In triangle ABC, T lies on $\overline{CA}$ past A such that $\angle TBA = (6💀)^\circ$ and $\angle TCB = (💀^2 - 4💀)^\circ$. If $BC = 4$ and $CA = 2$, find $[BAT]^2$.