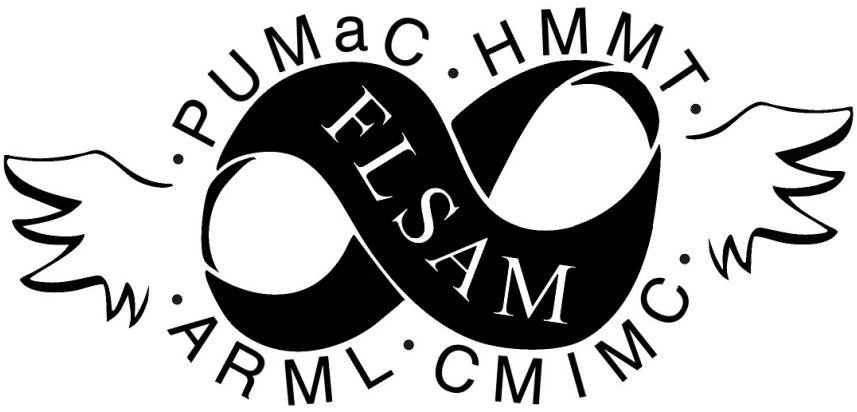
Florida Student Association of Mathematics



2023 Spooky Halloween Contest Team Round

October 2023

This is the team round, consisting of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty. As the name suggests, you may work together with your teammates to solve these problems.

*You will have **30 minutes** to complete the test. Good luck, and have fun!*

1. On each of eight tables, there sit eight pies of different flavors. There are eight pies of each flavor in the room. Each has eight slices. When each partygoer comes in, they randomly choose a slice of pie to eat. The probability the last two slices of pie eaten are of the same flavor, from the same table, or both can be expressed in simplest form as $\frac{p}{q}$. Find $p+q$.

2. Points B, O, N, E are chosen independently at random on the circumference of a circle. If the probability segments BO and NE intersect equals $\frac{a}{b}$ with $\gcd(a,b)=1$, find $a+b$.

3. A reveler in Candyland starts in the coordinate plane at $(0,7)$ hoping to attend a party at $(28,0)$. However, they must first visit any point on $y=1$ to withdraw chocolate coins from an ATM and then travel to $y=8$ to buy a costume. If they take the route of minimal distance, how many units must they travel before they reach the party?

4. Let triangle ABC have $AB=13$, $BC=15$, $AC=14$. Let I be the incenter of ABC, so that segments AI and BI intersect incircle ω at points S and T, respectively. The area of $\triangle SIT$ can be expressed as $\frac{a\sqrt{b}}{c}$ where a, b, c are integers, $\gcd(a,c)=1$, and b is not divisible by the square of any prime. Find $a+b+c$.

5. How many ordered pairs of integers (x,y) where $1 \leq x,y \leq 7$ satisfy $7 \mid x^3+xy+y^3+1$?

6. Let $MEOW$ be a parallelogram with point P in its interior such that $MP=3$, $EP=9$, $OP=8$, $WP=4$, and $\angle MPW+\angle EPO=180^\circ$. If the area of $MEOW$ can be written in simplest terms as $a\sqrt{b}$, find $a+b$.

7. Positive integers a and b are such that $\log_{\gcd(a,b)}(\operatorname{lcm}(a,b))$ is an integer greater than 2. Given that $a+b=35\gcd(a,b)$, find $\operatorname{lcm}(a,b)$.

8. A lonely witch begins a ritual on Halloween. During the first minute, she summons one toad. Each next minute, the toads summoned in the last minute each summon three more. Find the number of divisors of the smallest n such that after n minutes the number of toads is divisible by $2^2 \cdot 5^5 \cdot 13^3$.

9. Tom is trapped in a haunted house with doors labelled 1 to 999. A message on the wall tells him that the square of the exit's label ends with three identical digits. Compute the sum of all door numbers that could be the exit.

10. A magician has drawn a 5×5 square array of 25 evenly spaced dots. Confused by the ritualistic pentagram, he decides on a less involved shape. How many distinct non-degenerate quadrilaterals can he draw with vertices chosen from the array? (A quadrilateral is non-degenerate if no three vertices are collinear and it does not cross itself.)