Round 1

Name: _____

1. _____ **2.** _____

1. A triangle with side lengths 9, 10, and x has area $4x$. Compute the product of all possible values of x.

2. Let S, A, and M be digits such that

$$17 \cdot 27 \cdot 37 \cdot 47 \cdot 57 \cdot 67 = 3,048,SAM,619.$$

Find $100S + 10A + M$.

Round 2

Name: _____

3. _____ **4.** _____

3. Compute the number of 6-digit positive integers $\overline{ABCDEF}$ satisfying:

- $\overline{AB}$ is divisible by 4
- $\overline{ABCD}$ is divisible by 16
- $\overline{ABCDEF}$ is divisible by 64

4. Let $f(k, n)$ denote the number of positive integers less than n that can be represented as the sum of k (not necessarily distinct) powers of 2. Compute

$$f(4, 4096) + f(7, 4096)$$

Round 3

Name: _____

5. _____ **6.** _____

5. A random permutation of the divisors of 900 is chosen. Compute the expected number of pairs of consecutive elements such that the first element divides the second.

6. Consider right triangle TIM with $\angle I = 90^\circ$. Circle ω passes through I and is tangent to TM. Let ω intersect IM and TI again at E and F, respectively. Given that EF is parallel to TM and $\frac{TM}{EF} = 2025$, compute the sum of the possible values of $\frac{TI}{IM}$.

Round 4

Name: _____

7. _____ **8.** _____

7. On day n, Jayden, Sihwan, and Tiger each ate x_n, y_n, and z_n tacos, respectively, where $1 \leq x_n < y_n < z_n \leq 20$. After 90 days, they found out that they have eaten in such a way that x_n, y_n, z_n formed an arithmetic sequence every single day; furthermore, they realize they've eaten every possible triple (x, y, z) such that x, y, z are in arithmetic progression exactly once. Compute

$$\sum_{n=1}^{90} x_n y_n z_n.$$

8. Allen is opening a Temu factory with three assembly lines A, B, and C. Find the number of ways to assign his 10 distinguishable workers to the three lines such that within each assembly line the workers can pair up to check each other's work, with no worker left unpaired. Note that an assembly line may contain no workers.

Round 5

Name: _____

9. _____ **10.** _____

9. Suppose a, b, and c are real numbers such that

$$ab + bc + ca = 40 - a^2 = 45 - b^2 = 72 - c^2.$$

Compute $|abc|$.

10. Andrew is at the center of equilateral triangle P_0 with side length 1. At step n, he divides P_{n-1} into 4 congruent equilateral triangles and picks one of them at random to be P_n. Let C_n be the center of P_n. Compute the least real number r such that for all n, the expected value of $C_0C_n^2$ is at most r.

Round 6

Name: _____

11. _____ **12.** _____

11. Compute the sum of all distinct prime factors of 819604.

12. Triangle $\triangle PQR$ has $PQ = 13$, $QR = 14$, $RP = 15$. Let the midpoints of sides PQ, QR, RP be S, A, I. The incircle of triangle $\triangle SAI$ touches its sides AI, IS, SA at X, Y, Z; SX, AY, and IZ all intersect at a common point K. Find the area of $\triangle KRP$.

Round 7

Name: _____

13. _____ **14.** _____

13. Compute the number of ordered pairs (a, b) of elements from the set $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{100}\}$ such that ab is an integer.

14. For a positive integer n, define

$$f(n) = \left\lfloor \sum_{i=1}^{n} \gcd(i, n) \cos\left(\frac{2\pi i}{n}\right) \right\rfloor$$

Compute $f(f(f(2025)))$.