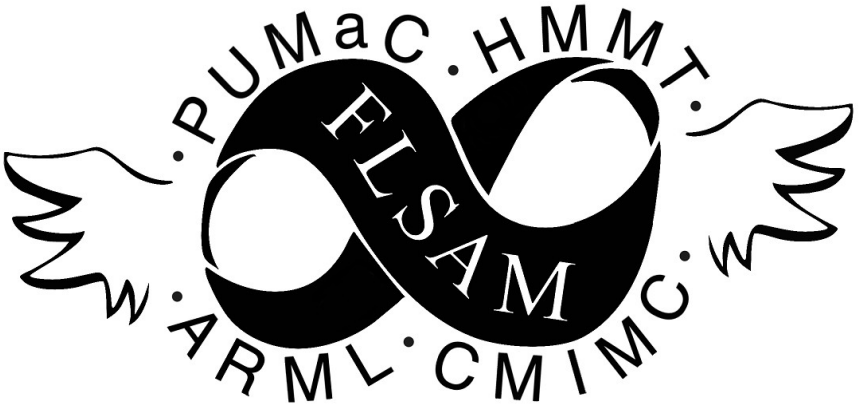
Florida Student Association of Mathematics



2024-2025 Florida Online Math Open Finals

Summer 2024-2025

Rules

1. You will have 12 hours to complete the contest. The test will be released at 12 PM on June 21st and submissions will be due by 12 AM on June 22nd.
2. This is a team event with teams of up to 4 students; collaboration with your teammates is encouraged. Discussion about the contest or its problems is not allowed between members of different teams or anyone not competing.
3. Each solution is graded out of 7 points following standard olympiad grading. Type out solutions using LaTeX.
4. Ties will be broken by quality of solutions.
5. Graphing calculators are permitted. The only drawing aids that are permitted are graph paper, rulers, compasses, and protractors.
6. Updates, including clarifications and errata, will be posted in the `#flomo-updates` channel of the Discord. Disputes that are answered by such updates will not be accepted. If you wish to ask a clarification question, contact either Aaryan Vaishya, Darsh Patel, or Tiger Ren privately through Discord.

1. Find all positive integers n for which there exists a prime $p > n$ such that for any positive integer m, there exists a positive integer r such that $n \equiv r^{2^m} \pmod{p}$.

2. How many pairs of 5-digit positive integers (a, b) are there such that a, b, and $a + b$ each have no 0's in their base 10 representations?

3. Consider acute triangle ABC with incircle ω and incenter I. Let its circumcircle be Ω, and let M be the midpoint of major arc BC. Let the tangents from M to ω intersect Ω again at D and E. Show that BC, DE, and IM are concurrent.

4. Let $s_b(a)$ be the sum of the digits of a when it is written in base b for any positive integers a and b with $b > 1$. Let $f(n)$ be the number of positive integers $m < 2^n$ with $s_4(m) = s_8(m)$. Show that there exists a unique positive real c such that for any $\varepsilon > 0$, there exists N for which

$$(c - \varepsilon)^n < f(n) < (c + \varepsilon)^n$$

holds for all $n > N$.

5. Let ABC be a triangle with circumcircle Γ and circumcenter O. The B-angle bisector intersects segment AC at P and Γ at $D \neq B$. The tangent to Γ at B intersects line AC at E, and line DE intersects Γ at $N \neq D$. Given that D, N, P, and O are concyclic, show that $NP = BP$.