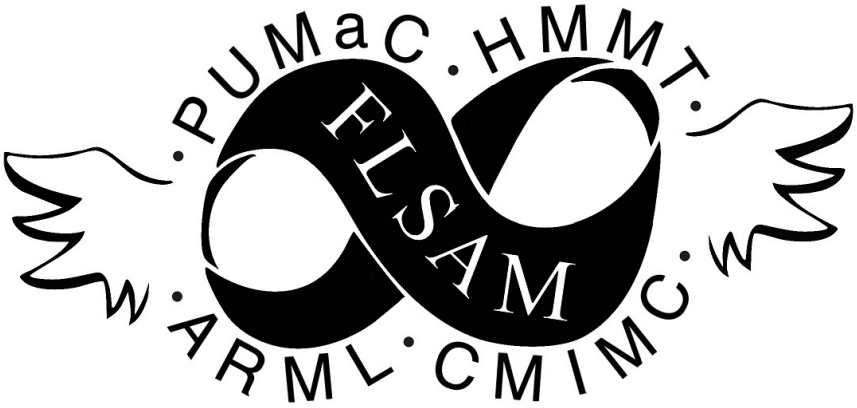
Florida Student Association of Mathematics



2025 HMMT Tryout

January 2025

Round 1: Algebra & Number Theory

Welcome to the **2025 FLSAM HMMT Tryout!** This is round one of three, consisting of six problems to be completed in 40 minutes. Scoring is simply based on correct answers; there is no penalty for wrong answers.

Answer Format:
All answers must be **fully simplified**; note that this is not necessarily the same as rules regarding answer formats for HMMT. Restrictions include but are not limited to fully reduced fractions, rationalizing denominators, and fully simplifying radicals. All reasonable fully simplified forms will be accepted in the case that multiple exist. These instructions apply for the other two rounds but will not be repeated.

1. A cubic polynomial $P(x)$ with rational coefficients is such that each of

$$P(\sqrt{2}) + \sqrt{2}$$
$$\sqrt{3}P(\sqrt{3}) + \sqrt{3}$$
$$P(\sqrt{5}) - \sqrt{5}$$
$$\sqrt{7}P(\sqrt{7}) - \sqrt{7}$$

are rational. Compute $P(1)$.

2. Positive reals x and y satisfy

$$\sqrt{x} + \sqrt{y} = 2$$
$$\sqrt{x+1} + \sqrt{y+1} = 3$$

Compute $x + y$.

3. Consider the polynomial $P(x) = x^{400} + 12x + 1$, having roots $r_1, r_2 \dots r_{400}$. Let $Q(x)$ be the monic polynomial whose roots are exactly $r_i^4 + 2$ for $i = 1, 2, \dots, 400$. Compute $Q(1)$.

4. Let $f(x) = x^2$. For an integer n, we let $d(n)$ be the least nonnegative integer k for which

$$\underbrace{f(f(\dots f}_{k+1\ f\text{'s}}(n)\dots)) \equiv \underbrace{f(f(\dots f}_{k\ f\text{'s}}(n)\dots)) \pmod{256}$$

Compute $\sum_{n=1}^{256} d(n)$.

5. Let a, b, c be the roots of the polynomial $x^3 - x^2 - 2x + 1$. Let S be the set of triples of nonnegative integers (i, j, k) with $i + j + k = 9$ and $ijk = 0$. Compute

$$\sum_{(i,j,k)\in S} a^i b^j c^k$$

6. Positive integers a_0, a_1, a_2, a_3, a_4 are such that

$$a_0a_0 + a_1a_4 + a_2a_3 + a_3a_2 + a_4a_1 = 187$$
$$a_0a_1 + a_1a_0 + a_2a_4 + a_3a_3 + a_4a_2 = 175$$
$$a_0a_2 + a_1a_1 + a_2a_0 + a_3a_4 + a_4a_3 = 184$$
$$a_0a_3 + a_1a_2 + a_2a_1 + a_3a_0 + a_4a_4 = 179$$
$$a_0a_4 + a_1a_3 + a_2a_2 + a_3a_1 + a_4a_0 = 175$$

Compute the sum of the possible values of $a_0 + a_1$.

Q1	Q2	Q3
Q4	Q5	Q6