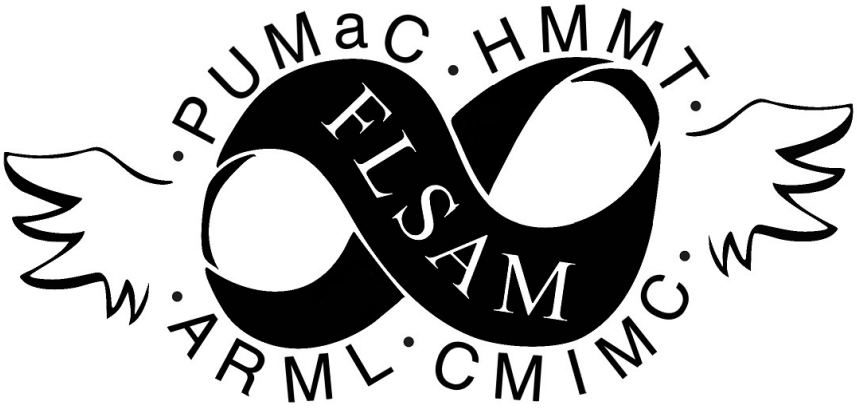
Florida Student Association of Mathematics



2025 HMMT Tryout

January 2025

Round 3: Combinatorics

This is round three of three, consisting of six problems to be completed in 40 minutes. Scoring is simply based on correct answers; there is no penalty for wrong answers.

1. Yusuf and Darsh are playing a game. First, Darsh rolls a die to receive a random number between 1 and 6 inclusive. Then, Yusuf rolls his rigged die that gives a random number between Darsh's number and 6 inclusive. Compute the expected value of the product of their numbers.

2. Compute the probability that 4 chords each formed from 2 randomly chosen points on a circle intersect at exactly 4 points and form a convex, non-self-intersecting quadrilateral.

3. Aaryan is cheating in darts. He takes a dartboard with 8 sectors separated by lines and removes 4 of the lines randomly. Compute the expected number of sectors after he does this.

4. Compute the number of permutations of $\{1, 2, 3, 4, 5\}$ such that the absolute difference between any pair of elements does not equal the absolute difference between their indices.

5. Positive real numbers x, y, z satisfying $x + y + z = 1$ are chosen randomly and uniformly. Compute the probability that

$$169yz + 196zx + 225xy \geq 64x + 49y + 36z$$

6. Four hooligans, Tiger, Darsh, Andrew, and Aaryan, live on an archipelago of 20 islands connected by one-way bridges. Yusuf places the bridges so that each island has a unique bridge leading towards it and a unique bridge leading away from it, with each possible placement of bridges being equally likely, and then randomly places the hooligans on different islands. A group of hooligans can have a *tea party* if they can, after traversing bridges, end up on the same island. Compute the probability that the four hooligans cannot have a tea party, but if Aaryan is excluded, then the remaining three can.

Q1	Q2	Q3
Q4	Q5	Q6