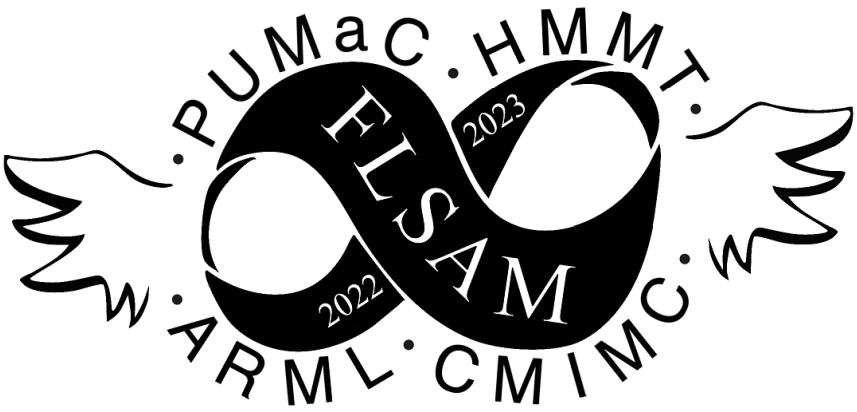
Florida Student Association of Mathematics



2024 Intro Meeting Test

October 2024

Welcome to the **2024 Intro Meeting Test**! This is the individual round, consisting of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty.

*You will have **60 minutes** to complete the test. Good luck, and have fun!*

1. Jason writes all the positive integers from 1 to 36, inclusive, on a blackboard. Daniel then erases two of his favorite distinct numbers. Afterwards, Jeffrey observes that the sum of the remaining numbers is equal to the product of the two numbers that Daniel erased. Determine the sum of the two numbers that Daniel erased.

2. Let $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ be the solutions to the equation $x^7 - x + 2 = 0$. Compute $x_1^7 + x_2^7 + x_3^7 + x_4^7 + x_5^7 + x_6^7 + x_7^7$.

3. A *geonacci* sequence is a sequence defined by the rule $a_{n+2} = a_{n+1}a_n$. Given $a_1 = 3, a_2 = 10$, compute the number of factors of a_8.

4. Sasha's favorite number satisfied the following conditions:

   - Exactly 4 distinct divisors of her favorite number have 4 factors.
   - Exactly 6 distinct divisors of her favorite number have 6 factors.

   Find the minimum possible value of Sasha's favorite number.

5. Given that $x^3 - ax^2 + bx - b = 0$ has 3 positive integer solutions, and that a and b are positive integers, compute the sum of all possible values of a.

6. Find the number of permutations of the word "FLORIDA" such that each letter appears at most two spots later than it does in FLORIDA.

7. Suppose a, b, and c are the solutions to $P(x) = x^3 + 4x^2 - 1 = 0$. Compute $\left(a + b + \frac{1}{c}\right)\left(b + c + \frac{1}{a}\right)\left(c + a + \frac{1}{b}\right)$.

8. Consider $\triangle ABC$ with $AB = 13, BC = 14, AC = 15$. Erect squares $ABDE, BCFG, ACHI$ outside the triangle, denote that the centers of the squares above as X, Y, Z, the area of $\triangle XYZ$ can be written in the simplest form of $\frac{m}{n}$ when $\gcd(m, n) = 1$ and $m, n \in \mathbb{N}$. Find $m + n$.

9. Find the number of ordered triples (a, b, c) of integers such that

$$a \leq b + c \leq a + 20$$
$$b \leq c + a \leq b + 20$$
$$c \leq a + b \leq c + 20$$

10. Consider a parabola with focus F and directrix ℓ. A circle ω with center O passes through F and intersects ℓ at A_1 and A_2, and segment $\overline{FO}$ intersects the parabola at B. Given $FA_1 = 5$, $FA_2 = 7$, $A_1A_2 = 6$, compute FB.