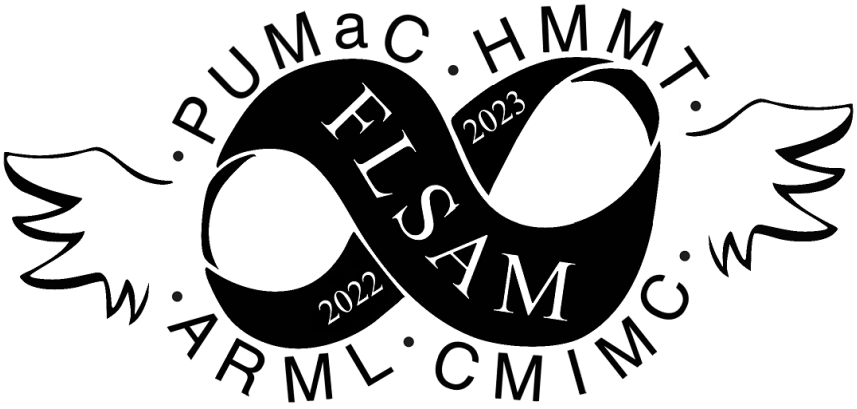
Florida Student Association of Mathematics



2024 FLSAM PUMAC Tryout

October 26-27, 2024

Round 4: Combinatorics

Welcome to the **2024 FLSAM PUMAC Tryout!** This is round four of four, each of which will consist of four problems to be completed in 30 minutes. Scoring is simply based on correct answers; there is no penalty for wrong answers. Good luck!

1. There are 3 singers performing. Each singer has a (possibly empty) set of the other singers that he wishes to perform after. Compute the number of possible values of x such that there are x ways to permute the singers that satisfy their wishes.

2. Let 10 points be randomly and independently chosen on the circumference of a circle. Compute the expected number of acute triangles formed by the points.

3. Let f be a function defined as

$$\begin{cases} f(1) = 1 \\ f(x) = \sum_{p|x} f(\frac{x}{p}) \quad x \neq 1, 30 \\ f(30) = 0 \end{cases}$$

   Where the sum is taken over all prime divisors p of x. Compute $f(1620000)$.

4. A complete graph on 5 vertices has its edges colored either red or blue randomly. Compute the probability that the graph has exactly one monochromatic triangle.