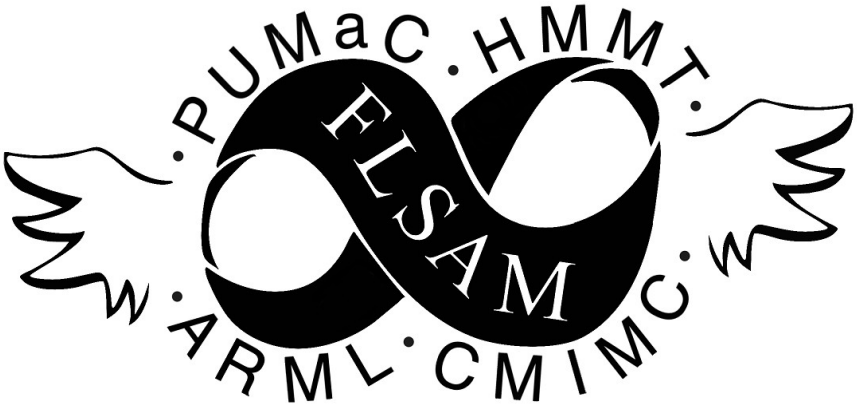
Florida Student Association of Mathematics



2025-2026 ARML Tryout

April 2026

Solutions

Answers

1. 20
2. 132
3. 216
4. $27\sqrt{3}$
5. 70
6. 2688
7. 138
8. 576
9. 6000
10. $\frac{625}{4}$
11. 5444
12. $13 + 6\sqrt{3}$
13. 96
14. 740005285

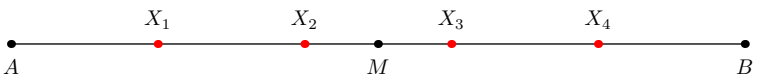
1. Let AB be a line segment with midpoint M. Distinct points X_1, X_2, X_3, and X_4 lie on segment AB such that

$$AX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4B.$$

Given that $MX_2 = 2$, compute the length AB.

Proposed by Jacob Khohayting

Answer: 20



Let AB have length ℓ. Then, $AM = \frac{\ell}{2}$ and $AX_2 = \frac{2\ell}{5}$. Hence,

$$MX_2 = AM - AX_2 = \frac{1}{2}\ell - \frac{2}{5}\ell = \frac{1}{10}\ell.$$

So, $\frac{\ell}{10} = 2$ and $\ell = \boxed{20}$.

2. Polynomials $A(x)$ and $B(x)$ with nonnegative integer coefficients are such that

$$A(x)B(x) - (x^2+1) = A(x) + xB(x)$$

for all real x. Compute the sum of all distinct possible values of $A(10)$.

Proposed by Darsh Patel

Answer: 132

We use Simon's Favorite Factoring Trick:

$$\begin{aligned} &A(x)B(x) - (x^2+1) = A(x) + xB(x) \\ \iff &A(x)B(x) - A(x) - xB(x) + x = x^2 + x + 1 \\ \iff &(A(x) - x)(B(x) - 1) = x^2 + x + 1 \end{aligned}$$

Then $A(x) - x$ must be a divisor of $x^2 + x + 1$; since $x^2 + x + 1$ is irreducible, we have 4 cases:

$$\begin{aligned} A(x) - x = 1 &\implies A(x) = x + 1 \\ A(x) - x = x^2 + x + 1 &\implies A(x) = x^2 + 2x + 1 \\ A(x) - x = -1 &\implies A(x) = x - 1 \\ A(x) - x = -(x^2 + x + 1) &\implies A(x) = -x^2 - 1 \end{aligned}$$

Only the first two have nonnegative coefficients, and give the possibilities $A(10) = 11, 121$. Their sum is $\boxed{132}$.

3. For a positive integer n, let $s_{10}(n)$ be the sum of the digits of n. Compute the product of all distinct possible values of $s_{10}(n+6) - s_{10}(n)$ over all two-digit positive integers n.

Proposed by Jacob Khohayting

Answer: 216

If there are no carries in the addition $n + 6$, then $s_{10}(n+6) = s_{10}(n) + 6$. If there are regroups, then each carry decreases the digital sum by 9. There can be at most 2 carries, which gives us the possibilities $6 - 9$ and $6 - 18$

for $s_{10}(n+6) - s_{10}(n)$, in addition to 6. We compute $6 \cdot (6-9) \cdot (6-18) = \boxed{216}$.

Note that $\dfrac{s_{10}(n+6) - s_{10}(n) - 6}{9}$ is the number of carries in the addition $n+6$; we remark that in general

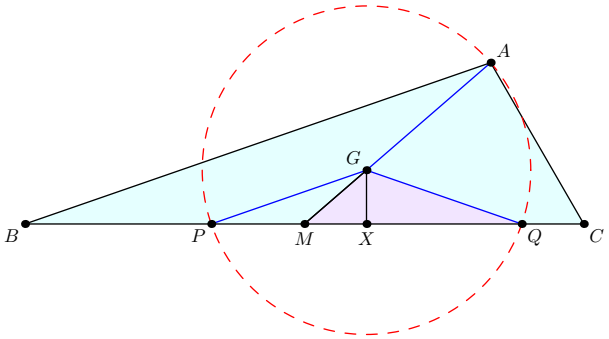
$$\frac{s_{10}(m+n) - s_{10}(m) - s_{10}(n)}{9}$$

is the number of carries in $m+n$.

4. Let ABC be a triangle with centroid G. The circle centered at G passing through A intersects segment BC at P and Q such that $BP = 6$, $PQ = 10$, and $QC = 2$. Compute the area of ABC.

Proposed by Jacob Khohayting

Answer: $27\sqrt{3}$



Let X be the foot of the perpendicular from G to BC. Then,

$$MX = BP + \frac{PQ}{2} - BM = 6 + 5 - 9 = 2.$$

Indeed, $XQ = \dfrac{PQ}{2} = 5$. Recall, $MG : GA = 1 : 2$. Let $MG = a$ and $GA = GQ = 2a$, then

$$GM^2 - MX^2 = GX^2 = GQ^2 - QX^2 \implies a^2 - 2^2 = GX^2 = (2a)^2 - 5^2.$$

Hence, $a^2 = 7$. Thus, $MG = \sqrt{7}$, then $GX = \sqrt{3}$, and so $d(A, BC) = 3\sqrt{3}$. Finishing,

$$[ABC] = \frac{1}{2} \cdot BC \cdot d(A, BC) = \boxed{27\sqrt{3}}.$$

5. An arithmetic sequence $s_1, s_2, s_3, \ldots$ of reals is such that

$$\frac{s_1}{2} = \frac{s_{s_1}}{11} = \frac{s_{s_{s_1}}}{53} \neq 0.$$

Compute s_{10}.

Proposed by Jacob Khohayting

Answer: 70

Let $s_1 = 2\ell$, then $s_{s_1} = s_{2\ell} = 11\ell$. Again, $s_{s_{s_1}} = s_{11\ell} = 53\ell$. Then,

$$\frac{2\ell - 1}{9\ell} = \frac{s_{2\ell} - s_1}{s_{11\ell} - s_{2\ell}} = \frac{11\ell - 2\ell}{53\ell - 11\ell} = \frac{3}{14}.$$

Hence, $28\ell - 14 = 27\ell$ and $\ell = 14$. So, $s_1 = 28$ and $s_{28} = 154$. This implies the common difference is

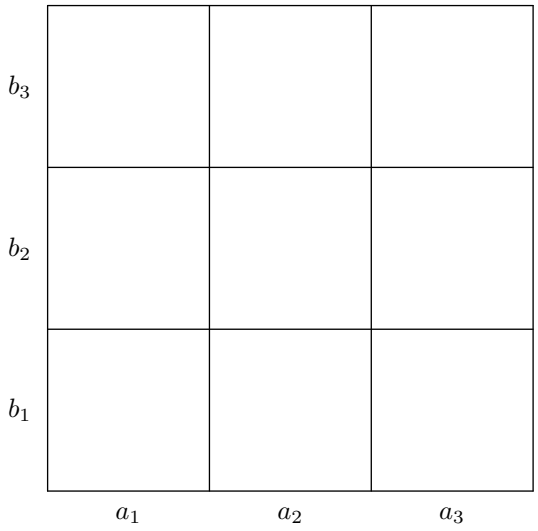
$$d = \frac{s_{28} - s_1}{28 - 1} = \frac{154 - 28}{28 - 1} = \frac{14}{3}.$$

Hence, $s_{10} = s_1 + 9d = 28 + 42 = \boxed{70}$.

6. Find the number of ways to place 9 standard six-sided dice onto a 3×3 grid such that for every pair of orthogonally adjacent dice, their touching faces have the same number.

Proposed by Jacob Khohayting

Answer: 2688



The key here is that once you place the values at the edges, the entire grid is fixed. This is due to the fact that the values of opposite faces of a die is always fixed. It suffices to determine for which values is such a configuration attainable.

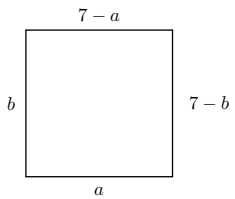
Denote by $s_1 = \{1, 6\}$, $s_2 = \{2, 5\}$, and $s_3 = \{3, 4\}$.

Claim 1. No such $1 \leq i, j, k \leq 3$ exist such that $a_i \in s_k$ and $b_j \in s_k$.

For the sake of contradiction, suppose $a_i \in s_k$ and b_j in s_k. Then, the die at the i-th column and j-th row must four faces whose values come from s_k. Notably, this is impossible. □

Claim 2. Given the values of the four faces which are not the top or bottom, then we can uniquely determine the die. This, of course, assumes the four given faces are actually attainable; that is, opposite faces sum to 7.

Suppose we know the following:



Rotate the die about the axis perpendicular to the sides of a and $7 - a$. Then, we will always be able to place b in the desired spot implying we can place $7 - b$ in the desired spot, as well. □

Now, we compute with the following two cases:

Case 1. Suppose $\{a\}$ and $\{b\}$ contains elements of only a single set s_i and s_j, respectively.

There are $3 \cdot 2$ ways to select the sets which appear. For each set $\{a\}$ and $\{b\}$, there are 2^3 ways to select elements. Hence,

$$(3 \cdot 2) \cdot 2^3 \cdot 2^3 = 384.$$

Case 2. Suppose $\{a\}$ contains elements of two sets s_i and s_j, and $\{b\}$ contains elements of a single set s_k.

There are $\binom{3}{2}$ to choose the placements of sets s_i. Then, there are 2^3 ways to place elements for $\{b\}$. But, for $\{a\}$, there are $4^3 - 2^3 - 2^3 = 48$ to place the elements. Hence,

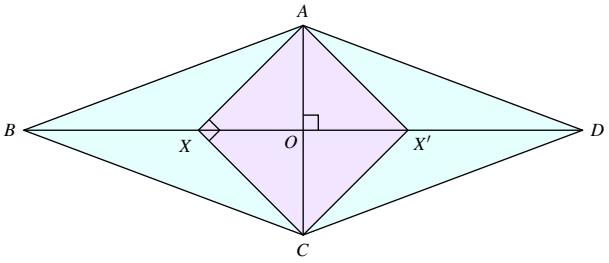
$$\binom{3}{2} \cdot 2^3 \cdot (4^3 - 2^3 - 2^3) = 1152.$$

We multiply by 2 for symmetry. Hence, the answer is $1152 \cdot 2 + 384 = \boxed{2688}$.

7. Let $ABCD$ be a rhombus. Suppose X is a point on segment BD such that $\angle AXC = 90^\circ$. Given that $BX = 20$ and $DX = 26$, compute the area of $ABCD$.

Proposed by Jacob Khohayting

Answer: 138



Let O be the center of $ABCD$. Then, $BO = \frac{20+26}{2} = 23$. Hence, $XO = 23 - 20 = 3$.
As $\angle AXC = 90^\circ$ and O is the midpoint of AC, then $AO = XO = CO = 3$. It follows

$$[ABCD] = \tfrac{1}{2} \cdot AC \cdot BD = \tfrac{1}{2} \cdot 6 \cdot 46 = \boxed{138}.$$

8. For a positive integer n, let $\mathcal{D}_n$ denote the set of positive divisors of n. Find the number of functions $f \colon \mathcal{D}_{2100} \longrightarrow \mathcal{D}_{2100}$ such that $f(ab) = f(a)f(b)$ for all integers a and b such that a, b, and ab are in $\mathcal{D}_{2100}$.

Proposed by Darsh Patel

Answer: 576

Prime factorize 2100 as $2^2 \cdot 3 \cdot 5^2 \cdot 7$. The multiplicativity condition $f(ab) = f(a)f(b)$ gives that $f(2^m 3^n 5^k 7^\ell) = f(2)^m f(3)^n f(5)^k f(7)^\ell$ for $m \leq 2, k \leq 2, n \leq 1, \ell \leq 1$. Now $f(2), f(3), f(5), f(7)$ are divisors of 2100 such that this expression is also always a divisor of 2100, which is equivalent to

$$f(2)^2 f(3) f(5)^2 f(7) \mid 2100$$

We can consider this divisibility with respect to each prime factor independently. For $p = 2, 3, 5, 7$, write

$$w_p = \nu_p(f(2)),\ x_p = \nu_p(f(3)),\ y_p = \nu_p(f(5)),\ z_p = \nu_p(f(7))$$

Then for $p = 3, 7$, we need

$$2w_p + x_p + 2y_p + z_p \leq 1$$

Then $w_p = y_p = 0$, and this has $\binom{3}{1} = 3$ solutions. For $p = 2, 5$, we need

$$2w_p + x_p + 2y_p + z_p \leq 2$$

This has 2 solutions where one of w_p, y_p are 1 and $\binom{4}{2} = 6$ solutions when $w_p = y_p = 0$ for 8 total. Then the total number of possibilities is $3^2 \cdot 8^2 = \boxed{576}$.

9. Find the unique positive integer N such that

- exactly 12 divisors of N are divisible by 12, and
- exactly 15 divisors of N are divisible by 15.

Proposed by Jacob Khohayting

Answer: 6000

Write

$$N = 2^a 3^b 5^c M$$

for M not divisible by $2, 3, 5$. The number of divisors of N divisible by 12 and 15 is

$$(a-1) \cdot b \cdot \tau(5^c M) = (a-1) \cdot b \cdot (c+1) \cdot \tau(M)$$

and

$$b \cdot c \cdot \tau(2^a M) = (a+1) \cdot b \cdot c \cdot \tau(M)$$

respectively, where $\tau(n)$ denotes the number of divisors of n. Note that we need $a \geq 2, b \geq 1, c \geq 1$. Writing

$$(a-1) \cdot b \cdot (c+1) \cdot \tau(M) = 12$$
$$(a+1) \cdot b \cdot c \cdot \tau(M) = 15$$

we see that $a+1$ is a divisor of 15 greater than 1. Then a is one of $2, 4, 14$.

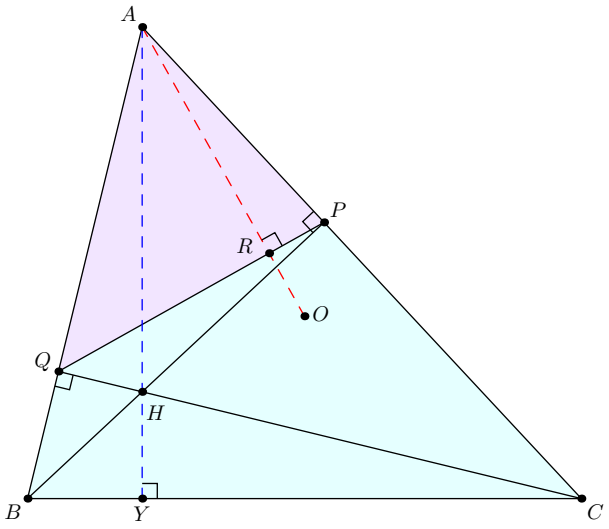
- Suppose $a = 2$. Then $b \cdot c \cdot \tau(M) = 5$. Since $5 \nmid 12$, we must have $(b, c, \tau(M)) = (1, 5, 1)$, which fails.
- Suppose $a = 4$. Then $b \cdot c \cdot \tau(M) = 3$. Manually checking each of the possibilities $(b, c, \tau(M)) = (3, 1, 1), (1, 3, 1), (1, 1, 3)$, only $(b, c, \tau(M)) = (1, 3, 1)$ works.
- Suppose $a = 14$. This fails because $13 \nmid 12$.

With $\tau(M) = 1$ yielding $M = 1$, we get $N = 2^4 \cdot 3^1 \cdot 5^3 \cdot 1 = \boxed{6000}$ as the only possibility.

10. Let ABC be an acute triangle with circumcenter O and altitudes BP and CQ such that $PQ = 12$ and $BC = 20$. Suppose that AO intersects PQ at R with $\frac{AR}{AO} = \frac{3}{4}$. Compute the area of ABC.

Proposed by Jacob Khohayting

Answer: $\frac{625}{4}$



Claim. $\triangle AQP \sim \triangle ACB$.

Evidently, $(BQPC)$ is cyclic as $\angle BQC = \angle BPC = 90^\circ$. Hence,

$$\angle AQP = 180 - \angle BQP = \angle PCB = \angle ACB,$$
$$\angle APQ = 180 - \angle BPQ = \angle PBC = \angle ABC.$$

The result follows. □

Claim. $AR \perp PQ$.

Notice, AH and AO are isogonal; that is, $\angle ABH = \angle ACO$. Recall, $AH \perp BC$ and $\triangle AQP \sim \triangle ACB$. So, under the mapping $ABC \mapsto APQ$, then $AY \mapsto AR$, and so $AR \perp PQ$, as desired. □

Let Y be the foot of the perpendicular from A to BC. Then,

$$\cos(A) = \frac{AQ}{AC} = \frac{PQ}{BC} = \frac{3}{5}.$$

Thus, $\sin(A) = \dfrac{4}{5}$. So,

$$R = \frac{BC}{2\sin(A)} = \frac{20}{2 \cdot \frac{4}{5}} = \frac{25}{2}.$$

Then,

$$AY = AR \cdot \frac{20}{12} = \left(AO \cdot \frac{3}{4}\right) \cdot \frac{20}{12} = AO \cdot \frac{5}{4} = \frac{125}{8}$$

Hence, $[ABC] = \dfrac{1}{2} \cdot 20 \cdot \dfrac{125}{8} = \boxed{\dfrac{625}{4}}$.

11. Find the unique four-digit positive integer $\overline{ABCD}$ with $A \neq 0$ such that 39 divides $\overline{A4BCD}$, $\overline{AB4CD}$, and $\overline{ABC4D}$.

Proposed by Jacob Khohayting

Answer: 5444

Notice,

$$\overline{A4BCD} - \overline{AB4CD} = 4 \cdot 900 - B \cdot 900 = (4 - B) \cdot 900 \equiv 0 \pmod{39}$$

Hence, $B \equiv 4 \pmod{13}$ which inevitably requires $B = 4$. Similarly, $C = 4$.

Then, $\overline{A444D}$ is divisible by 39. So,

$$\begin{aligned} 10000A + 4440 + D &\equiv 0 \pmod{13} \\ 3A + 7 + D &\equiv 0 \pmod{13} \\ 3A + D &\equiv 6 \pmod{13} \end{aligned}$$

Also, $A + 4 + 4 + 4 + D \equiv 0 \pmod{3}$ and so $A + D \equiv 0 \pmod{3}$. Either $3A + D = 6$, $3A + D = 19$, or $3A + D = 32$.

- If $3A + D = 6$, then $2A \equiv 0 \pmod{3}$ and so $A \equiv 0 \pmod{3}$. But, as $3A + D = 6$, then $A \leq 2$ forcing $A = 0$, impossible.
- If $3A + D = 32$, then $2A \equiv 2 \pmod{3}$ and so $A \equiv 1 \pmod{3}$. And so, $A = 7$ is the maximum possible value, but this is impossible, as well.

And so, $3A + D = 19$. So, $2A = 1 \pmod{3}$ and $A \equiv 2 \pmod{3}$. Clearly, $A = 2$ is too low and $A = 8$ is too high. Forcing $A = 5$ yields $D = 4$ which works. And so, $\boxed{5444}$ is indeed the unique desired value.

12. A degree 4 polynomial $P(x)$ with positive real coefficients is such that the coefficient of x^4 in $P(x)^n$ is n^4 for all positive integers n. Compute $P\left(\sqrt[4]{6}\right)$.

Proposed by Darsh Patel

Answer: $13+6\sqrt{3}$

Write $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. We find each term in the sum corresponding to the x^4 term in $P(x)^n$ based on how powers of x can multiply to x^4:

$$\begin{aligned} x^4: &\quad a_0^{n-1}a_4\binom{n}{1} \\ x^3\cdot x: &\quad 2a_0^{n-2}a_3a_1\binom{n}{2} \\ x^2\cdot x^2: &\quad a_0^{n-2}a_2^2\binom{n}{2} \\ x^2\cdot x\cdot x: &\quad 3a_0^{n-3}a_2a_1^2\binom{n}{3} \\ x\cdot x\cdot x\cdot x: &\quad a_0^{n-4}a_1^4\binom{n}{4} \end{aligned}$$

This is a result of, say, the Multinomial Theorem. Note that each of the constant leading coefficients correspond to the number of permutations of $(e_1, e_2, \ldots, e_k)$ for the decomposition $x^4 = x^{e_1}\cdot x^{e_2}\cdot\ldots\cdot x^{e_k}$. In order for the sum of these terms to be polynomial in n, we need $a_0 = 1$, whereupon their sum becomes

$$a_1^4\binom{n}{4} + 3a_2a_1^2\binom{n}{3} + (2a_3a_1 + a_2^2)\binom{n}{2} + a_4\binom{n}{1}$$

Recognizing that the binomial coefficient

$$\binom{n}{k} = \frac{n(n-1)\ldots(n-k+1)}{k!}$$

is a degree k polynomial in n, we can determine

$$n^4 = 24\binom{n}{4} + 36\binom{n}{3} + 14\binom{n}{2} + \binom{n}{1}$$

Matching coefficients, we have

$$\begin{aligned} a_1^4 &= 24 \\ 3a_2a_1^2 &= 36 \\ 2a_3a_1 + a_2^2 &= 14 \\ a_4 &= 1 \end{aligned}$$

The solution in positive reals to this system is

$$(a_1, a_2, a_3, a_4) = \left(\sqrt[4]{24}, \sqrt{6}, \frac{4}{\sqrt[4]{24}}, 1\right)$$

We now evaluate

$$P\left(\sqrt[4]{6}\right) = 1 + \sqrt[4]{24}\cdot\sqrt[4]{6} + \sqrt{6}\cdot\left(\sqrt[4]{6}\right)^2 + \frac{4}{\sqrt[4]{24}}\cdot\left(\sqrt[4]{6}\right)^3 + \left(\sqrt[4]{6}\right)^4 = \boxed{13+6\sqrt{3}}$$

13. Let $\mathcal{A} = (1,1,1,1,2,2,2,3,3,4)$. For a permutation of $\mathcal{A}$, let its *quintessentiality* be the number of entries of the permutation (before the final entry) that are at most the next entry. Let m be the minimum possible quintessentiality of a permutation of $\mathcal{A}$. Find the number of permutations of $\mathcal{A}$ with quintessentiality m.

Proposed by Jacob Khohayting

Answer: 96

Say two adjacent elements of the permutation are *quintessential pairs* if $a_i \leq a_{i+1}$. We want to minimize the number of quintessential pairs.

Claim. The minimum quintessentiality is 3.

Impossibility: Notice, at each value of 1 in which there is an element before it, this pair creates an quintessential pair. Therefore, there at least 3 quintessential pairs.

Attainability: Consider the construction below where each block contains integers in strictly decreasing order.

$$_\ _\ \dots\ _\ _\ 1 \mid _\ _\ \dots\ _\ _\ 1 \mid _\ _\ \dots\ _\ _\ 1 \mid _\ _\ \dots\ _\ _\ 1$$

This achieves a minimum of 3. □

It suffices to characterize which permutations have 3 quintessential pairs. The following is sufficient:

(a) the final element is 1 and

(b) between any two consecutive 1's, the array is *strictly* decreasing.

All permutations of such clearly work. It suffices to show this is necessary. Part (a) follows from impossibility. From there, we know there exists four blocks each ending with 1. If there exists any quintessential pair within the four blocks, then the quintessentiality is greater than 3. Hence, no such block can contain an quintessential pair. And so, the blocks are strictly decreasing proving Part (b).

It suffices to place the 2's, 3's, and 4's into different blocks. This yields

$$\binom{4}{3}\binom{4}{2}\binom{4}{1} = 4 \cdot 6 \cdot 4 = \boxed{96}.$$

14. Find the number of nonnegative integers less than $820 \cdot 10^6$ that cannot be written in the form $2^a + 820b + 1435c$ for nonnegative integers a, b, and c.

Proposed by Darsh Patel

Answer: 740005285

We first compute $\gcd(820, 1435) = 205 = 5 \cdot 41$. Thus, the expression can only ever achieve those residues classes modulo 205 achieved by powers of 2. The number of such residue classes is $\text{ord}_{205}(2)$. It is easy to find $\text{ord}_5(2) = 4$. Now, $\text{ord}_{41}(2)$ must be a divisor of 40. Then it either equals 40, or is a divisor of one of 8 or 20. We easily check $2^8 \not\equiv 1 \pmod{41}$. To find 2^{20} modulo 41, we first compute $2^{10} \equiv -1 \pmod{41}$. This yields that $2^{20} \equiv 1 \pmod{41}$, and since we already know $\text{ord}_{41}(2) \nmid 4$, we find $\text{ord}_{41}(2) = 20$. We now have

$$\text{ord}_{205}(2) = \text{lcm}(\text{ord}_5(2), \text{ord}_{41}(2)) = \text{lcm}(4, 20) = 20$$

We can also check that $2^{10} \equiv -1 \pmod{205}$, which will later be useful.

This immediately gives us $205 - 20 = 185$ residue classes modulo 205 that the expression cannot achieve, for

$$185 \cdot \frac{820 \cdot 10^6}{205} = 740 \cdot 10^6$$

values not achieved.

Consider the residue classes modulo 205 that can our expression can achieve. Each of $2^0, 2^1, \ldots, 2^{19}$ are the smallest values achieved by the expression within their respective residue classes. Let $r_{205}(n)$ denote the remainder of n upon division by 205. Then over all 20 residue classes, the values less than the smallest power of 2 in each residue class contribute

$$\left\lfloor \frac{2^0}{205} \right\rfloor + \left\lfloor \frac{2^1}{205} \right\rfloor + \cdots + \left\lfloor \frac{2^{19}}{205} \right\rfloor = \frac{2^0 - r_{205}(2^0)}{205} + \frac{2^1 - r_{205}(2^1)}{205} + \cdots + \frac{2^{19} - r_{205}(2^{19})}{205}$$

values not achieved. But $2^{n+10} \equiv -2^n \pmod{205}$, so $r_{205}(2^n) + r_{205}(2^{n+10}) = 205$. Combining pairs summing to 205, we compute this sum as

$$\frac{2^0 + 2^1 + \cdots + 2^{19}}{205} - 10 = \frac{(2^{10} - 1)(2^{10} + 1)}{205} - 10 = 5105$$

Now by Chicken McNugget Theorem, $820b + 1435c = 205 \cdot (4b + 7c)$ achieves all multiples of 205 at least $18 \cdot 205$. The second smallest powers of 2 in each of our 20 residues modulo 205, $2^{20}, 2^{21}, \ldots, 2^{39}$, are all clearly more than $18 \cdot 205$ more than their respective smaller power of 2. Thus, the inability of our expression to represent values in each residue class larger than the smallest power of 2 in that residue class comes from $4b + 7c$ not representing all small values. It is known that $4b + 7c$ fails to represent exactly $\frac{(4-1)(7-1)}{2} = 9$ nonnegative integers (which holds when 4 and 7 are replaced by any relatively prime nonnegative integers). As $2^{19} + 17 \cdot 205 < 820 \cdot 10^6$, we get the full 9 values missed in each residue class for $9 \cdot 20 = 180$ total.

The total number of values not represented by the expression is then

$$740 \cdot 10^6 + 5105 + 180 = \boxed{740005285}$$