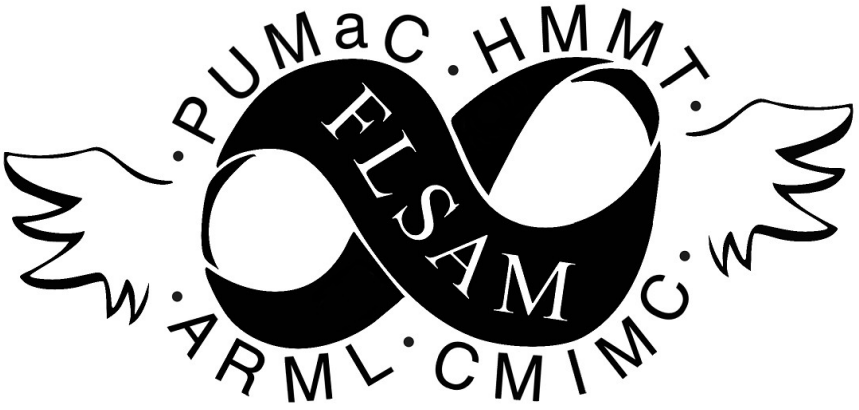
Florida Student Association of Mathematics



2025-2026 Florida Online Math Open

Winter 2025-2026

Rules

1. You will have 7 days to complete the contest. The test will be released at 7 PM on December 29 and submissions will be due by 7 PM on January 5.
2. This is a team event with teams of up to 6 students; collaboration with your teammates is encouraged. Discussion about the contest or its problems is not allowed between members of different teams or anyone not competing.
3. Each correct answer is worth 1 point. Incorrect or omitted answers are both worth 0 points.
4. Ties will be broken by looking at the most difficult question each team solved. If there is still a tie, the process will move on to the second most difficult question solved and so on. Difficulty is determined by the number of teams that solved a question, with fewer solves meaning a more difficult question.
5. Four function calculators are permitted, but graphing calculators and electronic programs are not. The only drawing aids that are permitted are rulers and compasses.
6. All answers are nonnegative integers. Any non-integer final value will be converted to a nonnegative integer.
7. Updates, including clarifications and errata, will be posted in the `#flomo-updates` channel of the Discord. Disputes that are answered by such updates will not be accepted. If you wish to ask a clarification question, contact either Jacob Khohayting or Darsh Patel privately through Discord.

1. Rectangle $ABCD$ has side lengths $AB = 24$ and $BC = 21$. Points P and Q lie on BC and DA, respectively, such that $AP = PQ = QC$. Find PQ.

2. Bob flips a fair coin until he gets 2 tails in total. Compute the expected number of times he flips the coin.

3. Jeffrey writes out the numbers 1 to 2025 from left to right. Between each pair of adjacent digits, Jeffrey either inserts a "$+$" sign or a "$-$" sign. Given that the result of Jeffrey's sum is negative, find the minimum number of "$-$" sign's Jeffrey could have used.

4. Find the number of six-digit positive integers N which satisfy the following properties:

- N is divisible by 2 and exactly two of its digits are 2; and
- N is divisible by 3 and exactly three of its digits are 3.

5. A real number α is chosen randomly and uniformly from the interval $[2, 5]$. Let p be the probability that 17 appears in the following sequence:

$$\lfloor \alpha \rfloor, \lfloor 2\alpha \rfloor, \lfloor 3\alpha \rfloor, \lfloor 4\alpha \rfloor, \ldots$$

If p can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n, compute $100m + n$.

6. Darsh plays a game starting with the number 0 and a set $S = \{1, 2, 4, \ldots, 512, 1024\}$. Every second, he adds an element of S, each with probability $\frac{1}{11}$, to his number. Let p be the probability that his new number is greater than 100 after 3 seconds. If p can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n, find $100m + n$.

7. Jason's favorite number is

$$\underbrace{12121212\ldots12121212}_{\text{2026 digits}}.$$

He inserts a 0 between some two consecutive digits such that the new number is divisible by 11. Over all 2025 possible insertions, find the number of possible ways he could have inserted a 0.

8. Let ABC be a triangle with orthocenter H such that M and N are midpoints of AB and AC, respectively. Suppose that $MH = 4$, $BH = 6$, and $CH = 8$. Find HN^2.

9. Arnav's favorite three-digit number is $\overline{ABC}$, where $A \neq 0$. When he interprets this as a base b number, he finds that it is divisible by 31_b for $b = 10, 13, 16$. Compute the sum of all distinct possible values of $\overline{ABC}$ (in base 10).

10. Find the number of ways to place 10 pieces, each colored either black or white, on a 5×5 grid such that

- each square contains at most one piece;
- no two white pieces are in the same row or column; and
- no two black pieces are in the same row or column.

11. Call a positive integer *n brainy* if it cannot be written in the form $67a + 68b + 69c$ for nonnegative integers a, b, and c. Compute the number of brainy positive integers.

12. Find the number of 5-tuples (F, L, S, A, M) of digits such that $F \cdot L$, $L \cdot S$, $S \cdot A$, and $A \cdot M$ all have units digit 6.

13. Let P be a point inside regular pentagon $ABCDE$ such that $\angle APE = 60^\circ$ and $\angle BPA = 66^\circ$. Find the degree measure of $\angle PDC$.

14. Let a, b, and c be non-zero real numbers such that

$$\frac{a^2c + c^2b + b^2a}{7} = \frac{a^2b + b^2c + c^2a}{11} = abc.$$

Compute $\dfrac{a^3}{b^3} + \dfrac{b^3}{c^3} + \dfrac{c^3}{a^3}$.

15. Let ABC be a triangle with incircle ω. The perpendicular bisector of AB is tangent to ω and an excircle is tangent to segment AB at D. Given that $AB = 20$ and $CD = 25$, find the square of the area of ABC.

(An *excircle* is the circle tangent to one side of the triangle and the extensions of the other two sides.)

16. Primes p and q are such that $\frac{3q+4}{p}$ and $\frac{4p+3}{q}$ are integers. Compute the sum of all distinct possible values of pq.

17. Let ABC be a triangle with points P and Q on segment BC with B, P, Q, C occurring in that order such that

$$\angle BAP = \angle PAQ = \angle QAC = 45^\circ.$$

Given that $BP = 40$ and $QC = 60$, compute the area of ABC.

18. For any positive integer n, let $f_3(n)$ be the largest power of 3 that divides n. For example, $f_3(7) = 1$ and $f_3(54) = 27$. Let $S = \sum_{n=1}^{\infty} \frac{f_3(n)}{n^2}$. Compute $\lfloor 1000S \rfloor$.

19. Roger chooses a random permutation $(S_1, S_2, \ldots, S_{2^{2025}})$ of the subsets of $\{1, 2, \ldots, 2025\}$. He now takes $A_1 = S_1$, and for $1 < k \leq 2^{2025}$ randomly chooses A_k to be either $A_{k-1} \cup S_k$ or $A_{k-1} \cap S_k$ with $\frac{1}{2}$ probability of either choice. Let E be the expected number of elements of $A_{2^{2025}}$. If E can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, compute $100m + n$.

20. There are 20 coins laid out on a table, with one of them showing tails and the rest showing heads. Every second, a coin is chosen at random and flipped over to reveal the other side. Find the expected number of seconds until all the coins show heads.

21. Let $ABCD$ be a trapezoid with bases AB and CD having side lengths $AB = 18$ and $CD = 90$. Let M and N be the midpoints of AD and BC, respectively. Points P and Q lying on MN with M, P, Q, N appearing in that order are such that

$$\angle ABQ = \angle PBC \quad \text{and} \quad \angle CDP = \angle QDA.$$

Given that $MP = 4$ and $QN = 18$, compute the square of the area of $ABCD$.

22. A positive integer n is called *lucid* if $n > 1$ and there do not exist positive integers $a, b, c, d < n$ for which

$$a^2b^4c^3d^5 \gcd(a^2 + b^4, c^3 + d^5) \cdot 2^a 4^b 3^c 5^d \operatorname{lcm}(2^a + 4^b, 3^c + 5^d)$$

is divisible by n. Compute the sum of all lucid n less than 300.

23. Positive real numbers $a_1, a_2, a_3, a_4, a_5, a_6$ and d_1, d_2, d_3 are such that

$$\begin{aligned}(a_1 + a_2)^2 &= a_3^2 + d_2^2 = a_6^2 + d_3^2 = 289\\ (a_3 + a_4)^2 &= a_5^2 + d_3^2 = a_2^2 + d_1^2 = 625\\ (a_5 + a_6)^2 &= a_1^2 + d_1^2 = a_4^2 + d_2^2 = 676.\end{aligned}$$

If $d_1 + d_2 + d_3$ can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n, compute $100m + n$.

24. Convex quadrilateral $ABCD$ has diagonals $AC = 72$ and $BD = 78$ which intersect at I. Let P, Q, and R be the circumcenters of triangles BIC, CID, and DIA respectively. It is given that $QI \perp AB$, $PI = 8$, and $RI = 32$. Compute the square of the area of $ABCD$.

25. Define a sequence $\{a_n\}_{n \geq 1}$ of reals by $a_1 = 2$ and

$$a_{n+1} = 2 - \sqrt{4 - a_n}$$

for $n \geq 1$. Let $P = \prod_{n=1}^{\infty} \frac{\sqrt{4 - a_n}}{2}$. Compute $\lfloor 1000P \rfloor$.

26. Jacob rolls 3^{2026} standard six-sided dice. For each integer r, let $p(r)$ be the probability that the sum of the values rolled is equivalent to r modulo 2026. From the set $\{0, 2, 4, \ldots, 2024\}$, Jacob chooses a and b for which $p(a)$ is minimal and $p(b)$ is maximal. Compute $a + b$.

27. A country contains the 5 cities A, B, C, D, and E. Between every pair of cities, a one-way road is built, whose direction is chosen at random. Let p be the probability that one can use the roads to travel from A to B, possibly traveling through intermediate cities. If p can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n, compute $100m + n$.

28. A nonzero polynomial $P(x)$ with integer coefficients with no integer roots is such that $P(n+5) - P(n)$ is not divisible by 25 for any integer n. For all positive integers n, let s_n be the largest nonnegative integer k such that 5^k divides $P(1)P(2)\dots P(n)$. Given that $s_{15625} = 7813$, compute the sum of all distinct possible values of s_{1250}.

29. Let $N = 10^{10}$. Integers $a_0, a_1, \dots, a_N$ are such that

$$a_0 n^{2025} + a_1 m(n+1)^{2025} + a_2 m^2 (n+2)^{2025} + \cdots + a_N m^N (n+N)^{2025} = 0$$

for all positive integers m and n with $m \leq 1000$. Let s be the smallest nonzero possible value of $|a_0 - a_1 + a_2 + \cdots + (-1)^N a_N|$. Compute the largest nonnegative integer k such that 2^k divides s.

30. Let $I = \displaystyle\int_1^{\sqrt{2}} \frac{\arctan(\sqrt{2-x^2})}{1+x^2}\, dx$. Compute $\lfloor I^{-2} \rfloor$.