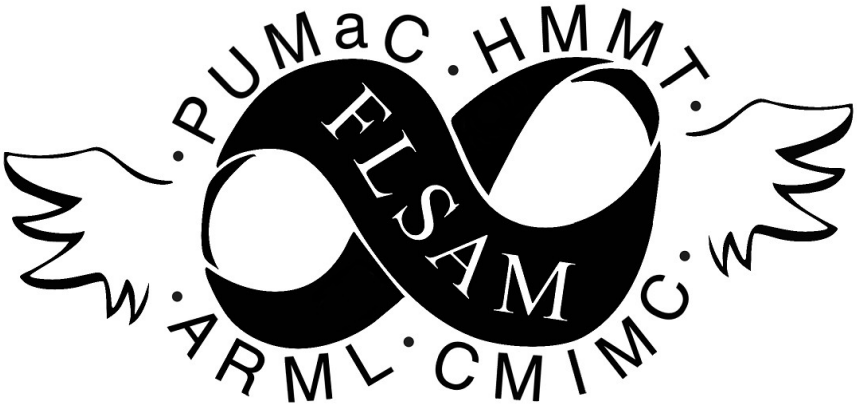
Florida Student Association of Mathematics



2025 Intro Meeting Test

September 2025

Welcome to the **2025 Intro Meeting Test**! This is the team round, consisting of 15 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty.

*You will have **60 minutes** to complete the test. Good luck, and have fun!*

1. Triangle $\mathcal{T}$ with area 2025 is drawn, and a point P is labeled in its interior. Jason, Daniel, and Jeffrey each draw a quadrilateral made up of vertices from $\mathcal{T}$ and P. To their surprise, they each drew a different figure! Find the sum of the areas of the quadrilaterals they drew.

2. Jason's favorite two-digit number is such that subtracting 2 or 4 from it results in a number that shares exactly one digit with the original, but subtracting 3 results in a number that shares no digits with the original. Find Jason's favorite number.

3. Ten people are lined up in a row, three of which are Aaryan, Ashvin, and Darsh. Given that Aaryan stands next to Ashvin, what is the probability Ashvin also stands next to Darsh?

4. Find the sum of the five smallest positive integers n such that $\gcd(n, 2100) = 15$.

5. A set of primes is called *complete* if the digits of its elements are $1, 2, \ldots, 9$ in some order. For example, the set $\{7, 83, 421, 659\}$ is complete. Compute the smallest possible sum of elements of a complete set.

6. Consider the function $f(x)$ satisfying

$$f(x+y) = f(x) + f(y)$$

for all real numbers x and y. Given $f(1) = 1$ and $f\left(\sqrt{2}\right) = \sqrt{3}$, compute $f\left(\frac{1}{2+\sqrt{2}}\right)$.

7. Triangle ABC has $\angle B = 90^\circ$ and centroid G. Points X and Y are on line AG such that $BX \perp AG$ and $CY \perp AG$. Suppose that $AB = 3$ and that $BC = 4$. Compute $AX - XY$.

8. Compute the number of five-digit numbers $\overline{ABCDE}$ such that none of

$$\overline{A}, \quad \overline{AB}, \quad \overline{ABC}, \quad \overline{ABCD}, \quad \overline{ABCDE}$$

are divisible by 3 or 10.

9. Consider trapezoid $ABCD$ where $AD \parallel BC$ and $AD < BC$. The angle bisectors $\angle A$ and $\angle D$ intersect at P such that $\angle APB = \angle CPD = 90^\circ$. Suppose that $BA = 10$, $AD = 7$, and $DC = 18$. Find the area of $ABCD$.

10. Each tile of a 3×3 chessboard is colored either white or black at random. Find the probability that a knight can move from the bottom left tile to the top right tile without changing colors or a knight can move from the bottom right tile to the top left tile without changing colors, but not both.

11. A four-digit positive integer $\overline{ABCD}$ is called *powerful* if A and C are nonzero and

$$\overline{ABCD} = \left(\overline{AB} + \overline{CD}\right)^2.$$

Find the sum of all powerful four-digit integers.

12. Alice and Bob play a game on n points spaced equally on a circle. Alice goes first, and on each turn the player draws a chord on the circle between some pair of the points. The first player to draw a chord intersecting an existing chord strictly in the interior of the circle loses. Compute the sum of all positive integers n between 3 and 100 inclusive such that Alice wins when the game is played optimally.

13. Let ABC be a triangle with $AB = 13$, $BC = 14$, $CA = 15$, and circumcenter O. Suppose AO intersects the circumcircle of $\triangle BOC$ at $X \neq O$. Find the length of AX.

14. Positive reals a, b, and c satisfy the following system of equations:

$$a^2 = bc - 8$$
$$b^2 = ca + 1$$
$$c^2 = ab + 10.$$

Find $a + b + c$.

15. Triangle $\triangle$ in $\mathbb{R}^3$ has projections $\triangle_{xy}$, $\triangle_{yz}$, and $\triangle_{zx}$ onto the xy, yz, and zx planes, respectively. Given that the areas of $\triangle_{xy}$, $\triangle_{yz}$, and $\triangle_{zx}$ are 5, 8, and 44 respectively, find the area of $\triangle$.