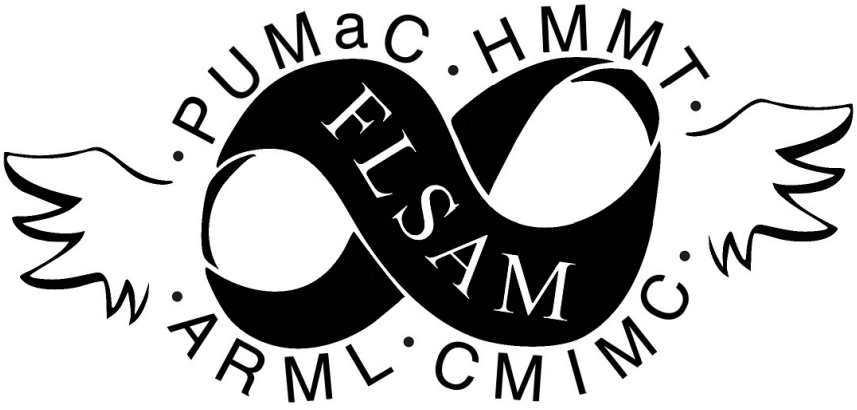
Florida Student Association of Mathematics



2025 FLSAM PUMAC Tryout

October 25-26, 2025

Round 2: Number Theory

Welcome to the **2025 FLSAM PUMAC Tryout!** This is round two of four, each of which will consist of four problems to be completed in 30 minutes. There is no penalty for wrong answers. Good luck!

1. Find the smallest positive integer n such that the number of positive integers k at most n with $\gcd(k, n) = 1$ is exactly $\frac{16}{67}n$.

2. Find the smallest positive integer n such that

$$3\left\lfloor\frac{n}{3}\right\rfloor - 4\left\lfloor\frac{n}{4}\right\rfloor = 7\left\lfloor\frac{n}{7}\right\rfloor - 8\left\lfloor\frac{n}{8}\right\rfloor = 11\left\lfloor\frac{n}{11}\right\rfloor - 12\left\lfloor\frac{n}{12}\right\rfloor = 1.$$

3. A sequence $\{a_n\}_{n\geq 1}$ of positive integers is defined recursively by $a_1 = k$ and

$$\begin{cases} a_{n+1} = \frac{1}{2} \cdot a_n & \text{if } a_n \equiv 0 \pmod 2 \\ a_{n+1} = a_n + 77 & \text{if } a_n \equiv 1 \pmod 2 \end{cases}$$

for all n. Find the sum of all values of k for which $a_6 = a_1$.

4. Aaryan calculates the product $2025 \cdot 2026$ by starting with $S = 0$ and adding 2025 to S until he gets $S = 2025 \cdot 2026$. Compute the average number of regroups/carries he does per addition over all 2026 additions.

[For example, the addition $9843 + 417$ involves 3 regroups/carries, occurring at the ones, hundreds, and thousands digits.]

Q1	Q2	Q3	Q4