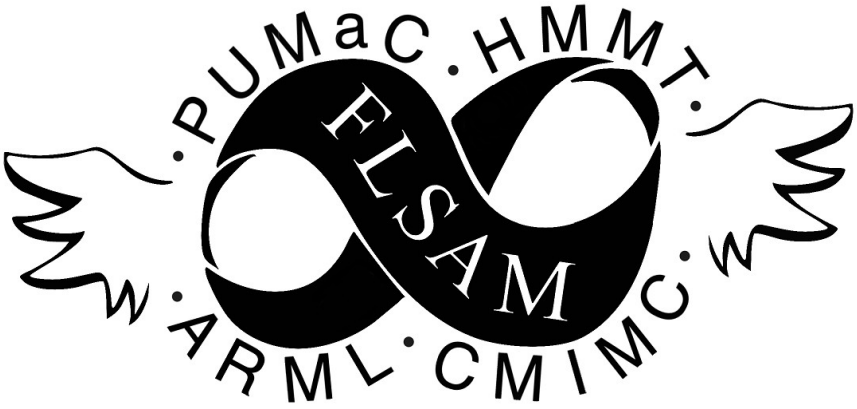
Florida Student Association of Mathematics



2026 Intro Meeting Test Solutions

September 19 – 20, 2026

Welcome to the **2026 Intro Meeting Test**! This is the team round, consisting of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and all answers are single values.

*You will have **50 minutes** to complete the test. Good luck, and have fun!*

Answer Key

1. $\frac{36}{41}$
2. 1008
3. $\frac{5039}{5040}$
4. $\frac{104}{5}$
5. $\frac{1}{12}$
6. 4073
7. 149
8. 74
9. $24\sqrt{34}$
10. $\frac{\pi^2}{20}$

1. Ben and Jerry each open an ice cream shop stocked with the same number of tubs. Ben has sold six times as many tubs as Jerry, while Jerry has seven times as many tubs left as Ben. Compute the fraction of his stock Ben has sold.

Proposed by Jacob Khohayting

Answer: $\dfrac{36}{41}$

Suppose Ben has sold $6t$ tubs, then Jerry has sold t tubs. If Jerry has $7s$ tubs remaining, then Ben has s tubs remaining. Equating, then

$$t + 7s = 6t + s \implies 5t = 6s \implies t = \frac{6s}{5}.$$

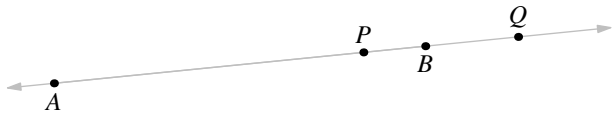
The fraction that Ben has sold is

$$\frac{6t}{s+6t} = \frac{36/5}{1+36/5} = \boxed{\frac{36}{41}}.$$

2. Two distinct points on line AB are each five times as far from A as from B. If the distance between the two points is 420, compute the length AB.

Proposed by Jacob Khohayting

Answer: 1008



We have three scenarios:

- Suppose a point X is to the left of A. Note that

  $$XA < XB < 5 \cdot XB.$$

  Hence, it is impossible for XA to be five times as far from XB, in this scenario.
- Suppose a point X is between A and B. Then, $XA = t$ and $XB = AB - t$. Hence,

  $$XA = 5 \cdot XB \implies t = 5 \cdot AB - 5t \implies t = \tfrac{5}{6} \cdot AB.$$

  This refers to the point P above.
- Suppose a point X to the right of B. Then, $XA = t$ and $XB = t - AB$. Hence,

  $$XA = 5 \cdot XB \implies t = 5t - 5 \cdot AB \implies t = \tfrac{5}{4} \cdot AB.$$

  This refers to the point Q above.

So, as $PQ = 420$, then

$$420 = PQ = \frac{5}{4} \cdot AB - \frac{5}{6} \cdot AB = \frac{10}{24} \cdot AB.$$

Solving implies $AB = 42 \cdot 24 = \boxed{1008}$.

3. Let N be a positive rational number such that, when N and $10080N$ are written in lowest terms, the sum of the numerator and denominator is the same. Compute the largest possible value of N.

Proposed by Jacob Khohayting

Answer: $\dfrac{5039}{5040}$

Suppose $N = \frac{a}{b}$ for integers a and b where $\gcd(a, b) = 1$. Let $\gcd(b, 10080) = g$. Then,

$$\frac{10080a}{b} \to \frac{10080a/g}{b/g},$$

when written in simplest form. So,

$$\frac{ga}{g} + \frac{gb}{g} = \frac{10080a}{g} + \frac{b}{g} \implies (g-1)b = (10080-g)a \implies \frac{a}{b} = \frac{g-1}{10080-g}.$$

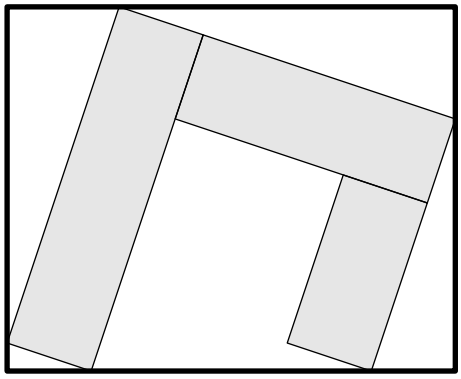
Equivalently,

$$\frac{g-1}{10080-g} = -1 + \frac{10079}{10080-g}.$$

This is maximized when $10080 - g$ is minimized. As g takes all possible divisors of 10080 (other than 1 and 10080), take $g = 5040$ which yields

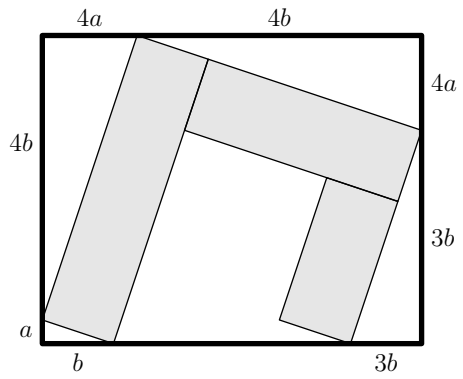
$$\frac{g-1}{10080-g} = \boxed{\frac{5039}{5040}}.$$

4. Rectangular blocks with dimensions 1×2, 1×3, and 1×4 are arranged inside a rectangle, as shown below. Compute the area of the rectangle.



Proposed by Jacob Khohayting

Answer: $\dfrac{104}{5}$



Notice each triangle in the figure is similar. As a result, we can say each right triangle is similar to one with legs a and b but with hypotenuse 1. Upon labeling lengths, we can comparing opposite sides yielding

$$a + 4b = 4a + 3b \implies b = 3a.$$

Due to the Pythagorean Theorem, $a^2 + b^2 = 1$, so $a = \frac{1}{\sqrt{10}}$ and $b = \frac{3}{\sqrt{10}}$. Therefore, we can express the area of the entire rectangle as

$$(4b + a)(4a + 4b) = \left(\frac{13}{\sqrt{10}}\right)\left(\frac{16}{\sqrt{10}}\right) = \boxed{\frac{104}{5}}.$$

5. The equation

$$x^{\log_2 x} = 12x^{12}.$$

has two distinct solutions a and b. Compute $a^{\log_2 b}$.

Proposed by Joshua Khohayting

Answer: $\frac{1}{12}$

Solution: Let $x = 2^t$. Substituting, it follows that

$$\begin{aligned} x^{\log_2 x} &= 12x^{12} \\ 2^{t^2} &= 12 \cdot 2^{12t} \\ t^2 - 12t - \log_2 12 &= 0. \end{aligned}$$

If $a = 2^u$ and $b = 2^v$, it follows that u and v are the solutions to the quadratic. By Vieta's,

$$uv = -\log_2 12.$$

Therefore,

$$a^{\log_2 b} = 2^{u \cdot v} = 2^{-\log_2 12} = \boxed{\frac{1}{12}}.$$

Notably, the quadratic has two distinct real solutions as the discriminant

$$(-12)^2 - 4 \cdot 1 \cdot (-\log_2(12)) \geq 144$$

is positive.

6. Alice and Bob play a game with a pile of N sticks, taking turns with Alice going first. On each turn, a player must remove at least one stick, but no more than half of the sticks currently in the pile. A player who cannot make a legal move loses. Compute the least positive integer N for which Alice can guarantee a win by removing exactly 2026 sticks on her first move.

Proposed by Abhishek Chand

Answer: 4073

We claim that a pile with N sticks is a losing position if and only if $N = 2^k - 1$ for some non-negative integer k.

For $k = 0$, then $N = 0$, which is trivially a losing position as no moves can be made. For $k = 1$, then $N = 1$, a player cannot remove any sticks because $1 > \lfloor \frac{1}{2} \rfloor = 0$, so no legal move exists and $N = 1$ is also a losing position.

Assume for all $j < k$, any pile of size $2^j - 1$ is a losing position. Consider a pile of size $N = 2^k - 1$. A player must remove x sticks, where

$$1 \leq x \leq \left\lfloor \frac{2^k - 1}{2} \right\rfloor = 2^{k-1} - 1$$

The remaining number of sticks $R = (2^k - 1) - x$ satisfies

$$2^{k-1} \leq R \leq 2^k - 2$$

Notice that R lies strictly between two consecutive losing positions ($2^{k-1} - 1$ and $2^k - 1$). Thus, any move from $2^k - 1$ leaves a position that is not of the form $2^j - 1$.

The second player can then remove $y = 2^{k-1} - x$ sticks to reduce the pile to $R - y = 2^{k-1} - 1$, which is a losing position for the first player. To confirm this is a legal move, we verify $1 \leq y \leq \lfloor \frac{R}{2} \rfloor$:

- Since $x \leq 2^{k-1} - 1$, we have $y = 2^{k-1} - x \geq 1$.
- We require $2y \leq R \implies 2(2^{k-1} - x) \leq 2^k - 1 - x \implies x \geq 1$, which holds true.

Therefore, from any position $N = 2^k - 1$, every move leads to a winning position for the opponent, making $2^k - 1$ a losing position.

Since all positions not of the form $2^k - 1$ are winning positions, a player starting at such N can win by reducing the pile to $2^k - 1$ for the largest integer k such that $2^k - 1 < N$.

Alice wants to win by removing exactly $x = 2026$ sticks on her first move, leaving a pile of size $R = N - 2026$. For this move to guarantee a win, R must be a losing position, so $R = 2^k - 1$ for some integer k.

Alice's move must also satisfy the game's constraint $x \leq \lfloor \frac{N}{2} \rfloor$:

$$2026 \leq \left\lfloor \frac{N}{2} \right\rfloor \implies N \geq 2 \times 2026 = 4052$$

To minimize N, we express N as $N = R + 2026 = (2^k - 1) + 2026$. We find the smallest integer k such that $N \geq 4052$:

$$(2^k - 1) + 2026 \geq 4052 \implies 2^k + 2025 \geq 4052 \implies 2^k \geq 2027$$

The smallest power of 2 satisfying $2^k \geq 2027$ is $2^{11} = 2048$ (since $2^{10} = 1024$). Thus, the minimum positive integer N is:

$$N = (2^{11} - 1) + 2026 = 2047 + 2026 = \boxed{4073}$$

7. Let a and b be positive real numbers such that the smallest positive integers r and s for which ab^r and $a^s b$ are integers are 16 and 28, respectively. Compute the smallest positive integer t such that $a^t b^t$ must be an integer.

Proposed by Jacob Khohayting

Answer: 149

Let m and n be positive integers. Then, consider

$$(ab^r)^m (a^s b)^n = a^{m+ns} b^{rm+n},$$

which must be an integer. As $m + ns = rm + n$, then $n(s-1) = m(r-1)$. Let $g = \gcd(s-1, r-1)$. Write $s - 1 = gx$ and $r - 1 = gy$. Then, we require $n = cy$ and $m = cx$ for some constant positive integer c. To minimize $m + ns$, we wish to minimize c which occurs when $c = 1$. Therefore,

$$\begin{aligned} t = \min(m+ns) = x + y(gx+1) &= gxy + x + y \\ &= \frac{g^2xy + gx + gy}{g} \\ &= \frac{(gx+1)(gy+1) - 1}{g} \\ &= \frac{rs-1}{gcd(s-1, r-1)}. \end{aligned}$$

In this case, plugging in $r = 16$ and $s = 28$ yields $t = \boxed{149}$.

For completeness, one can verify $a = 2^{16/447}$ and $b = 2^{-1/447}$ has a smallest possible value for t of 149. In particular, the smallest values for which

$$\begin{aligned} 447 \mid (16 - r), \\ 447 \mid (16s - 1). \end{aligned}$$

occurs when $r = 16$ and $s = 28$, evidently. Indeed,

$$447 \mid (16t - t) = 15t \implies 149 \mid 5t.$$

For this to hold, then $149 \mid t$. The smallest value is therefore 149, as desired.

8. For a finite set $\mathcal{S}$ of positive integers, define the *quiddity* of $\mathcal{S}$ to be the smallest positive integer that cannot be expressed as the sum of elements of any subset of $\mathcal{S}$. Compute the number of sets of the minimum possible number of elements with a quiddity of 2026.

Proposed by Roger He & Jacob Khohayting

Answer: 74

The maximum quiddity of a set with a 1 element is 2. Suppose that for a set with n elements, the maximum quiddity is 2^n achievable with the set

$$\mathcal{S} = \{2^0, 2^1, \ldots, 2^{n-1}\}.$$

Then, for a set with $n + 1$ elements, we can toss in the element which cannot be obtained, 2^n, yielding a quiddity of 2^{n+1}. Suppose there exists a set $\mathcal{S}'$ with a quiddity of $Q > 2^{n+1}$ and the $|S'| = n + 1$. If any element $e \in S'$ is less than 2^n, then $S \setminus \{e\}$ should have quiddity

$$Q - e > 2^{n+1} - 2^n = 2^n.$$

Note that $1 \in S'$, so such an element e exists. Hence, the maximum quiddity is 2^{n+1} for an $n+1$ element set, completing the induction. It follows a set with quiddity of 2026 must contain exactly 11 elements.

Consider the n element set $\mathcal{S} = \{2^0, 2^1, \ldots, 2^{n-1}\}$. Similarly, consider the n element set

$$S' = \{s(0), s(1), \ldots, s(n-1)\},$$

where $s(i) < s(i+1)$ for all $0 \leq i \leq n-2$. Suppose there exists some integer $0 \leq k \leq n-1$ for which $2^k < s(k)$ and k' be the least of all such k satisfying this property. Note that

$$s(0) + s(1) + \ldots + s(k-1) \leq 2^0 + 2^1 + \cdots + 2^{k-1} = 2^k - 1 < s(k) + 1.$$

As a result, the quiddity would be at most 2^k. If $n = 11$, then the quiddity would be at most 1024 if there exists some element for which $2^k < s(k)$. Hence, all elements must satisfy $s(k) \leq 2^k$. In particular,

$$s(i) \leq 1 + s(0) + s(1) + s(2) + \cdots + s(i-1).$$

Indeed, $s(0) + s(1) + \cdots + s(10) = 2025$. Let

$$p(i) = s(0) + s(1) + \cdots + s(i)$$

be the ith prefix sum. Note $p(10) = 2025$. Note that the two things we proved previously imply $s(i+1) \leq p(i) + 1$. Define $d(i) = (2^i - 1) - p(i)$, the error of $p(i)$, and $e(i) = p(i-1) + 1 - s(i) \geq 0$, the error of $s(i)$. Note that $d(0) = 0$ and $d(i) = 2d(i-1) + e(i)$. Hence,

$$d(10) = 1024e(0) + 512e(1) + \cdots + 32e(5) + 16e(6) + 8e(7) + 4e(8) + 2e(9) + e(10).$$

As $p(10) = 2025$, then $d(10) = 2047 - 2025 = 22$. Hence,

$$22 = 16e(6) + 8e(7) + 4e(8) + 2e(9) + e(10).$$

There exists a bijection from the number of solutions to this equation and the number of partitions of 22 into powers of 2 where order matters. Notably, the recursion for the number of partitions of n is

$$\begin{aligned} b_{2m+1} &= b_{2m} \\ b_{2m} &= b_{2m-1} + b_m. \end{aligned}$$

The first equation comes the fact that every partition of an odd number $2m+1$ must contain 1, which upon removal yields a partition of $2m$. The second comes from the fact that if a partition has 1, then excluding it results in a partition of $2m-1$ but if a partition doesn't include 1, then we can divide by 2 to each element yielding a partition of n. We can easily find the recursion:

n	$b(n)$
0	1
1	1
2	2
3	2
4	4
5	4
6	6
7	6
8	10
9	10
10	14
11	14

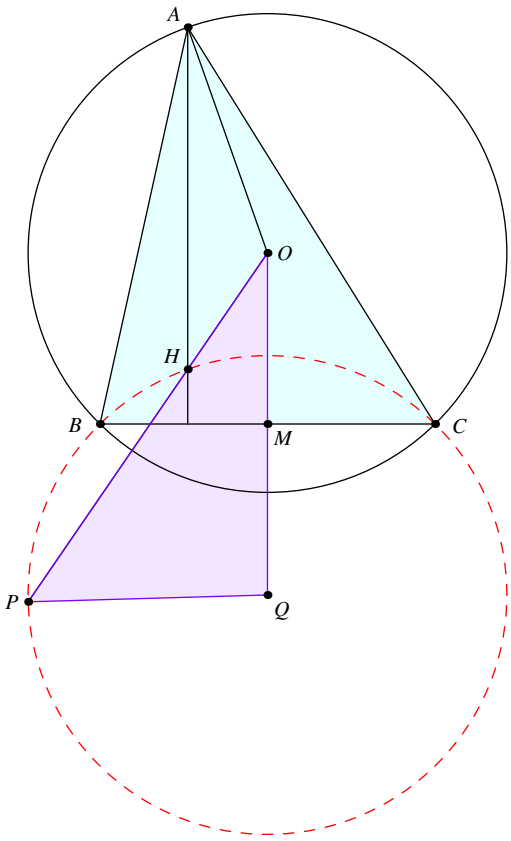
n	$b(n)$
12	20
13	20
14	26
15	26
16	36
17	36
18	46
19	46
20	60
21	60
22	74

Therefore, the answer is $\boxed{74}$.

9. Acute triangle ABC with orthocenter H and circumcenter O satisfies $AH = 20$ and $AO = 14$. Line OH intersects the circumcircle of BHC at P such that $\frac{PH}{HO} = 2$. If the circumcenter of BHC is Q, compute the area of OPQ.

Proposed by Arnav Bhatia

Answer: $24\sqrt{34}$



Recall, due to the Orthocenter Reflection Lemma, $(ABC) \cong (BHC)$. In addition, as $AH = 2 \cdot OM$, then

$$OQ = 2 \cdot OM = AH = 20.$$

Let $OH = x$. Then, $OP = OH + HP = x + 2x = 3x$. By Power of Point at O, then

$$3x^2 = x \cdot 3x = OH \cdot OP = OQ^2 - R^2 = 20^2 - 14^2 = 204.$$

Hence, $x^2 = 68$ and $x = 2\sqrt{17}$.

Let R be the foot of the perpendicular from O to AH. Set $AR = y$. $RH = 20 - y$, and $RO = z$. Then, due to the Pythagorean Theorem,

$$y^2 + z^2 = AO^2 = 196,$$
$$(20 - y)^2 + z^2 = OH^2 = 68.$$

Subtracting implies $400 - 40y = 128$ implying $40y = 528$ or $AR = y = \frac{66}{5}$. As a result,

$$z = RO = \sqrt{AO^2 - AR^2} = \sqrt{14^2 - \left(\tfrac{66}{5}\right)^2} = \frac{4\sqrt{34}}{5}.$$

Now, as $OP = 3 \cdot OH$, then

$$3 \cdot d(O, AH) = d(P, OQ) \implies d(P, OQ) = \frac{12\sqrt{34}}{5}.$$

Therefore,

$$[OPQ] = \frac{1}{2} \cdot OQ \cdot d(P, OQ) = \frac{1}{2} \cdot 20 \cdot \frac{12\sqrt{34}}{5} = \boxed{24\sqrt{34}}.$$

10. For a positive integer n, define $f(n)$ to be the remainder when the total number of prime factors of n, counting multiplicity, is divided by 2. Compute

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^2}.$$

Proposed by George Paret

Answer: $\dfrac{\pi^2}{20}$

Suppose the prime factorization of n can be written as

$$n = p_1^{e_1} \cdot p_2^{e_2} \dots p_k^{e_k}.$$

Define the function $d(n) = \sum e_i$. Then, $f(n) \equiv d(n) \pmod 2$.

Now, consider the function $p(n) = 1 - 2f(n)$. This function returns 1 when $f(n) = 0$ or when the number of primes is even. On the other hand, this function returns -1 when there is an odd number of primes and this means that $p(n) = (-1)^{d(n)}$.

Let a and b be relatively positive integers. Let $a = p_1^{e_1} \cdots p_k^{e_k}$ and $b = q_1^{r_1} \cdots q_\ell^{r_\ell}$. Since $\gcd(a, b) = 1$, the primes p_i and q_j are all distinct. Thus

$$ab = p_1^{e_1} \cdots p_k^{e_k} q_1^{r_1} \cdots q_\ell^{r_\ell}.$$

Therefore,

$$\begin{aligned} d(ab) &= e_1 + \cdots + e_k + r_1 + \cdots + r_\ell \\ &= d(a) + d(b). \end{aligned}$$

Hence,

$$\begin{aligned} p(ab) &= (-1)^{d(ab)} \\ &= (-1)^{d(a)+d(b)} \\ &= (-1)^{d(a)}(-1)^{d(b)} \\ &= p(a)p(b). \end{aligned}$$

Therefore, $\frac{p(x)}{x^2}$ is a multiplicative function.

Now, we can rewrite our sum as

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^2} = \sum_{n=1}^{\infty} \frac{\left(\frac{1-p(n)}{2}\right)}{n^2} = \frac{1}{2}\left(\frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{p(n)}{n^2}\right).$$

We will consider just the summation. Because the function is multiplicative, we can write

$$\sum_{n=1}^{\infty} \frac{p(n)}{n^2} = \prod_{q} \sum_{i=0}^{\infty} \frac{p(q^i)}{q^{2i}}.$$

Note that

$$\frac{p(q^i)}{q^{2i}} = \frac{(-1)^{d(q^i)}}{q^{2i}} = \frac{(-1)^i}{q^{2i}} = \left(\frac{-1}{q^2}\right)^i.$$

Therefore, we can express the sum as

$$\prod_{q} \sum_{i=0}^{\infty} \left(\frac{-1}{q^2}\right)^i = \prod_{q} \frac{1}{1+\frac{1}{q^2}} = \prod_{q} \frac{1-q^{-2}}{1-q^{-4}} = \frac{\frac{1}{\zeta(2)}}{\frac{1}{\zeta(4)}} = \frac{\zeta(4)}{\zeta(2)} = \frac{\frac{\pi^4}{90}}{\frac{\pi^2}{6}} = \frac{\pi^2}{15}.$$

Thus, the answer is

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^2} = \frac{1}{2}\left(\frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{p(n)}{n^2}\right) = \frac{1}{2}\left(\frac{\pi^2}{6} - \frac{\pi^2}{15}\right) = \boxed{\frac{\pi^2}{20}}.$$