Florida Student Association of Mathematics

2022-2023 Introduction Meeting — October 2022

Individual Round

Welcome to the **FLSAM Intro Meeting Individual Round!** This event consists of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point.

*You will have **60 minutes** to complete the test. Good luck, and have fun!*

1. Andrew chooses two positive divisors of 14^{14} and notes that they sum to 357. What number does he get when he computes the absolute difference of these two numbers?

2. In a unit square $ABCD$, points P and Q are chosen on the perimeter such that $PQ = 1$. As P and Q vary, the midpoint of PQ traces out a curve. The area enclosed by this curve can be expressed as $a - \frac{\pi}{b}$, for positive integers a and b. Compute $a^2 + b^2$.

3. Given that
$$\lceil x \rceil^2 - 12\lfloor x \rfloor + 20 = 0,$$
find the sum of all distinct possible values of $\lfloor 2x \rfloor$. (Here $\lfloor x \rfloor$ denotes the greatest integer that is at most x, and $\lceil x \rceil$ denotes the least integer that is at least x.)

4. A frog jumps on lily pads starting at lily pad 1 such that from lily pad n the frog can jump to either lily pad $n+1$ or lily pad $n+2$. Denote A_n as the number of paths the frog can take to reach lily pad n. Compute $A_1 + A_2 + \cdots + A_{10}$.

5. Jessica fills the 64 squares of an 8×8 table with distinct positive integers from 1 to 66 in a way such that the sum of each row is divisible by 17 and the sum of each column is divisible by 21. What is the sum of the two positive integers she omits?

6. Aaron has a standard 52 deck of cards. He randomly draws 2 of them, and they're both aces! Now, he randomly distributes the remaining cards by giving 5 cards each to 10 friends. The probability that no one gets more than one ace can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m + n$.

7. Alex is tired of the boring geometric series, and so he decides to add some spice by developing a probabilistic geometric series! The first term of this series in 1, and every successive term is the previous term divided by 2 or the previous term divided by 3, each occurring with probability $\frac{1}{2}$. The expected value of the sum of this series can be expressed as $\frac{m}{n}$, for relatively prime positive integers m, n. Compute $m + n$.

8. Jack has 6 presents for 6 children, with each present being for exactly 1 child and no 2 presents being for the same child. However he forgot to write down who each present is for after wrapping them! If he randomly gives each child a gift, the probability that more than 1 child gets their intended gift can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. What is $m + n$?

9. Let $a_1, a_2, \ldots, a_n$ be an arithmetic sequence with first term 1, and let $g_1, g_2, \ldots, g_n$ be a geometric sequence with first term 1, where all terms of both sequences are integers. Let $s_1, s_2, \ldots, s_n$ be the sequence with $s_k = a_k + g_k$. What is the smallest integer $k > 1$ such that s_k cannot equal 2022?

10. Let $ABCD$ be a trapezoid with $\overline{BC} \parallel \overline{AD}$ and $AB = 13$, $BC = 14$, $CD = 15$. Given that the B-median in $\triangle ABC$ and the C-median in $\triangle BCD$ intersect on $\overline{AD}$, find the area of $ABCD$.