Florida Student Association of Mathematics

2022 Spooky Halloween Contest Boss Fight

October-November 2022

As you finish placing your decorations in the graveyard, the Ghost House Of Scary Troubles produces 3 Mega Ghosts and their minions to stop you! To defeat the Ghost House, you must work with your team to accumulate as many points as possible; each point deals one point of damage to the House, which needs 7$\times$ [the number of people in your team] points to be destroyed. Note that you are permitted to communicate with your team.

*You will have **30 minutes** to complete the test. Good luck, and have fun!*

Set 1

1. **[4]** KitKats can be approximated as trapezoidal prisms while York candies can be approximated as cylinders. KitKats consist of 4 prisms, with the trapezoidal cross section having bases of lengths 12mm and 16mm, and a height of 10mm. The lengths of the prisms are 90mm. Yorks have heights of 10mm and radii of 24mm. The ratio of the volume of a KitKat to a York can be expressed as simple fraction $\frac{m}{n\pi}$. Compute $m + n$.

2. **[4]** A scary number frightens Andrew if, in base 10, it is less than 1024 and greater than 0, but in base 16, the sum of its digits is divisible by 5. Naturally, Aaron writes down every possible scary number. How many numbers does Aaron write?

3. **[4]** Skeletons numbered 1 through 10 are racing around a circular track starting at the 0 meter mark. Skeleton n goes at a speed of n meters per second and the circumference of the track is 99 meters. A skeleton trap activates at the 50 meter mark every 20 seconds. If the skeleton's run around the track forever, how many skeletons are eventually trapped?

4. **[4]** Compute the sum of all possible positive integers $\overline{SKUL}$ satisfying $\overline{SKUL} = \overline{UL}^2$, where S, K, U, L are (not necessarily distinct) digits not all equal to 0.

Set 2

5. **[6]** There is a 60% chance that Gerber sees a skull on Halloween. If he sees a skull, he loses focus and only collects half as many candies as if he had not seen a skull. The expected number of candies Gerber collects is 56. If he takes a different route so that there is only a 30% chance of seeing a skull, how many pieces of candy can Gerber now expect to collect? Assume that the number of candies collected given that Gerber sees a skull and the number given he doesn't see a skull remain the same as before.

6. **[6]** Jessica is scared of all numbers expressible in the form

$$\frac{abc}{a+b+c},$$

where a, b, c are positive integers. What is the largest integer less than 2022 that she is not scared of?

7. **[6]** Over all pairs of integers (x, y) with $0 \leq x, y \leq 2022$ satisfying

$$y^{\log_3(y^4)} = x^{\log_{\sqrt{3}}(y^{10}) - \log_9(x^{50})},$$

what is the sum of all possible values of $x + y$?

8. **[6]** Find the sum of the three distinct prime factors of $6^8 - 6^5 + 1$.

Set 3

9. **[8]** There exists a function $f(x) = x^4 - 4x^3 + 3x^2 - 2x + 1$. Let the roots of $f(x)$ be r_1, r_2, r_3, r_4. There exists another function $g(x)$ with 6 roots of the form $r_i + r_j$ where $0 < i < j \leq 4$. Given that $g(x)$ is a monic polynomial of degree 6, find the coefficient of the x^4 term in $g(x)$.

10. **[8]** n $\triangle ABC$, let M, N denote the midpoints of $\overline{AB}$, $\overline{BC}$, respectively. Suppose that the circumcircle of $\triangle AMN$ is tangent to $\overline{BC}$ and intersects $\overline{AC}$ a second time at $D \neq A$. Given that $AN = 9$ and $BC = 12$, find AD^2.

11. **[8]** Raphael is in a haunted house and he can't get out! The resident Zombie gives Raphael the following problem: $2^{22} + 1$ has 3 distinct prime factors, denoted as p_1, p_2, and p_3, where $p_1 \leq p_2 \leq p_3$. In order to get our of the haunted house, Raphael has to take p_2 steps. How many steps should he take?

12. **[8]** 5 people in costumes play a game. In a round, each person randomly chooses to unmask one other person, and those who have been revealed lose. The game ends when no more legal moves can be made, i.e. no one can unmask another player still in the game. Given the probability that there is exactly one winner equals $\frac{m}{n}$ for relatively prime positive integers m, n, find $m + n$.