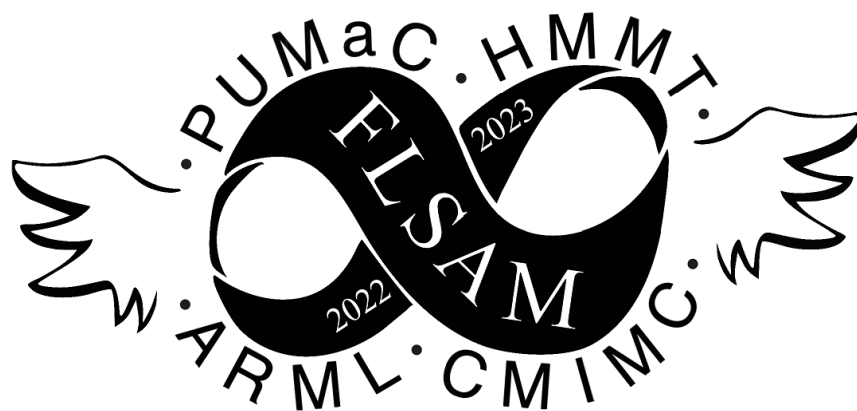


Florida Student Association of Mathematics



2022 Spooky Halloween Contest Individual Round

October-November 2022

Welcome to the **2022 FLSAM Spooky Halloween Contest!** This is the individual round, consisting of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty.

*You will have **40 minutes** to complete the test. Good luck, and have fun!*

1. Wichita arranges her 300 toads along a hallway. The weights of the 300 toads form an arithmetic sequence. The first 100 toads weigh 100 units in total and the combined weight of all the toads is 700 units. What is d^{-1} , where d is the common difference in the arithmetic sequence of weights?
2. Andrew is giving out treats. For any child, there is a $\frac{2}{3}$ chance he gives out nothing and a $\frac{1}{3}$ chance he gives a treat. A child is tricked when the child before them gets a treat, but they get nothing. What is the expected number of tricked children if Andrew meets 100 children?
3. A ghost's favorite three-digit number plus the sum of its digits is a multiple of 99. Find the sum of all possible values of this number.
4. Let Y and R respectively be the feet of the altitudes from A and C of acute triangle $\triangle SCA$ such that $AC = 25$, $AY = 24$, and $CR = 20$. Find the area of $\triangle CRY$.
5. Ramez and Yuhan go to a fair to celebrate Halloween. There is a restaurant selling pumpkin pies and hot dogs. There are 10 pies and 6 hot dogs, each stored in its own non-distinguishable box. If Ramez and Yuhan choose 8 boxes randomly, the probability Ramez has more hot dogs than Yuhan can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n . Compute $m + n$.
6. Aaron the sorcerer's favorite function is $f(x) = \frac{1}{3+3^x}$. Given that

$$f\left(\frac{1}{2022}\right) + f\left(\frac{2}{2022}\right) + \cdots + f\left(\frac{4042}{2022}\right) + f\left(\frac{4043}{2022}\right)$$

can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n , compute $m + n$.

7. A quirky pumpkin carver draws a coordinate axis on his pumpkin. He draws the square mouth with a side of the square on the line $y = x + 2$ and the opposite side of the square on the curve $x = y^2$. What is the maximum possible area of the mouth square?
8. Initially, there is a singular bone in a graveyard. Every minute, the number of bones in the graveyard is multiplied by a random factor of 2022. If the number of bones in the graveyard is a multiple of 2022, all the bones combine to form a skeleton. The expected number of minutes it takes for a skeleton to form can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n . Compute $m + n$.
9. Call an ordered pair of positive integers (a, b) *spooky* if it satisfies

$$\gcd(a, b)^2 = \text{lcm}(a, b) + 2023.$$

What is the number of spooky pairs of positive integers?

10. Tiger is performing a magic trick at a Halloween fair! He has 97 numbers $\frac{49}{1}, \frac{49}{2}, \dots, \frac{49}{97}$ and he asks you to keep replacing two numbers, say a and b , with $2ab - a - b + 1$. Then, he predicts what the last number is! As always, he is right. What was his prediction?