Florida Student Association of Mathematics

2022 Spooky Halloween Contest Team Round

October-November 2022

This is the team round, consisting of 10 problems in the subjects of algebra, geometry, combinatorics, and number theory. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty. As the name suggests, you may work together with your teammates to solve these problems.

*You will have **30 minutes** to complete the test. Good luck, and have fun!*

1. Yuhan eats an unusual snack for Halloween that consists of a sliced cucumber with initial mass 300g saturated in Spooky Essence^{TM}, and a sliced cucumber with initial mass 250g saturated in pumpkin spice latte. The cucumbers are 96% water, while the Spooky Essence^{TM} is 70% water and the pumpkin spice latte is 90% water. Assuming cucumbers are saturated when they absorb 50% of their mass, the fraction of the mass of Yuhan's snack that was water is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

2. Andrew is making Halloween friendship bracelets. For every bracelet, he puts 5 orange beads, 7 black beads, and 11 white beads in a random order on a wire. If the leftmost bead on the wire has place 1, and every place to the right increases by 1, what place is the first orange bead expected to be?

3. Giving out candy is very troublesome for Darius. Each time he opens the door to give out candy

- with a probability of $\frac{4}{5}$, he returns to his chair
- with probability $\frac{3}{20}$, the doorbell rings on his way back to his chair and he opens the door again
- and with probability $\frac{1}{20}$, he either gives up giving out candy or runs out and rests for the rest of the night.

What is the expected number of times he sits back down in his chair before he gives up or runs out and rests for the night?

4. To ward away a sphinx, Alex must find the sum of all nonnegative integers n where $9n+16$ and $16n+9$ are both positive squares. What answer should Alex give to survive?

5. The number $16^{32}+128$ has a unique 3-digit prime factor. Compute this prime factor.

6. Call a pair of distinct, positive real numbers (a,b) *compatible* if the following statement is true:

$$a+\frac{b}{a+\frac{b}{a+\dots}}=b+\frac{a}{b+\frac{a}{b+\dots}}.$$

Over compatible (a,b), the minimum of a^2+4b^2 is $\frac{m}{n}$ for coprime, positive integers m,n. Compute m^2+n^2.

7. Algebra problems are scary right? When solved over reals,

$$\left(x^2+1\right)\left(y^2+6\right)\left(z^2+24\right)=96xyz$$

has solutions at $(x_1,y_1,z_1),(x_2,y_2,z_2)\dots(x_n,y_n,z_n)$. Compute $\prod_{i=1}^{n} x_iy_iz_i \pmod{1000}$.

8. A ghost goes trick-or-treating in an infinite neighborhood numbered $3,6,9,\dots$ Each house it knocks on will only give it candy if the number of previous houses who have treated the ghoul (with candy!) divides its house number, though the first house (number 3) offers it candy regardless. Find the number of pieces of candy it will have collected after visiting 2022 houses.

9. Quadrilateral $GHST$ is inscribed in a circle with center O and radius 22. Given that $\overline{GT}\parallel\overline{HS}$, $\angle HOS=90^\circ$, and pentagon $GHOST$ is convex, what is the difference between the maximum and minimum possible areas of $GHOST$?

10. In concave equilateral hexagon $HAUNTS$, we have $\angle A=108^\circ$ and $\angle H=\angle U=96^\circ$. Compute $\angle T$ given that it is less than 180°.