Florida Student Association of Mathematics

2022-2023 Everything Tryout

January 21-22, 2023

Round 2: Geometry

1. Let $ABCD$ be a rhombus with side length 6. Given that the angle bisectors of $\angle ABD$ and $\angle ACD$ intersect on $\overline{AD}$, compute $\mathcal{A}^2$, where $\mathcal{A}$ is the area of $ABCD$.

2. Consider $\triangle ABC$ with $AB = 4$ and $AC = 8$. Given that the length of the angle bisector of $\angle BAC$ contained inside the circumcircle of $\triangle ABC$ is 12, find BC^2.

3. Concave pentagon $ABCDE$ has side lengths all equal to 1 and $\angle D > 180^\circ$. Additionally, $AC = BE = \sqrt{1+\sqrt{3}}$. Compute $\angle D$, in degrees.

4. The parabolas $6y = x^2 - 100$ and $8x = y^2 - 100$ intersect in quadrants 2 and 4 of the coordinate plane at A and B, respectively. The perpendicular bisector of $\overline{AB}$ passes through a unique lattice point (a, b). Compute $a^3 + b^3$.

5. In unit square $ABCD$, equilateral triangles ABE and ABF are erected outside and inside the rectangle, respectively. Line CF intersects the circumcircle of DEF again at $S \neq F$, and line DF intersects the circumcircle of CEF again at $T \neq F$. Find the integer closest to ST^3.

6. Triangle $\triangle ABC$ has side lengths $AB = 13$, $BC = 14$, and $AC = 15$, as well as circumcircle ω. The sides of $\triangle ABC$ partition ω into 4 regions: 1 triangle, and 3 circular segments bounded by a side opposite from a vertex, and a minor arc. Call these regions R_A, R_B, and R_C based on which vertex the side is opposite of (e.g. R_A is bounded by side $\overline{BC}$). Circles ω_A, ω_B, and ω_C are the circles inscribed in R_A, R_B, and R_C, respectively, with maximal area.

   The external tangents of ω_B and ω_A intersect at B_1, and the external tangents of ω_C and ω_A intersect at C_1. The circle centered at B_1 passing through C and the circle centered at C_1 passing through B intersect inside $\triangle ABC$ at A_1. Compute AA_1^2.