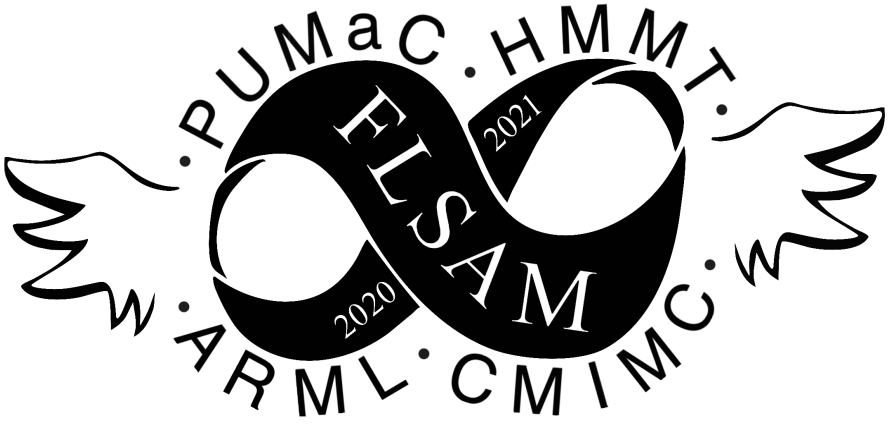
Florida Student Association of Mathematics



2020 Spooky Halloween Contest Team Round

October 2020

This is the team round, consisting of 10 problems in the subjects of algebra, geometry, combinatorics, and number theory. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty. As the name suggests, you may work together with your teammates to solve these problems.

*You will have **30 minutes** to complete the test. Good luck, and have fun!*

1. Spooky Serum^{TM} is back, and Divij has two potions, each containing only Spooky Essence^{TM} and pumpkin spice latte: Potion A containing 1% Spooky Essence^{TM} and Potion B containing 2% pumpkin spice latte. Connor's patented Spooky Serum^{TM} requires a mixture of 73.75% Spooky Essence^{TM} and 27.25% pumpkin spice latte. Divij mixes some amount of each potion together to form Spooky Serum^{TM} . What is the ratio of Potion B to Potion A in the final mixture?

2. A 3-digit number is *very spooky* if it is two times the number formed by reversing its digits. Unfortunately, there aren't any very spooky numbers in base 10. However, there is exactly one very spooky number base 8 — what is it in base 10?

3. Saathvik the spider is bored of making his web, and instead graphs a polynomial $P(x)$ on the coordinate plane using his silk. This line passes through the interior (not including boundary) of eight unit squares in the 4×4 square with vertices $(0,0), (4,0), (4,4), (0,4)$: All four corner unit squares, and four unit squares inside the square with vertices $(1,1), (3,1), (3,3), (1,3)$. What is the minimum degree of $P(x)$?

4. Iris wants to carve pumpkins *in style.* So, she places the two pumpkins at $(5,3)$ and $(1,5)$ inside a reflective square room with vertices $(0,0), (7,0), (7,7), (0,7)$. Standing at $(0,0)$, she fires her ultra-spooky laser cannon in a straight line, bouncing off the edges of the square. If it hits a vertex, it reverses direction. What is the minimum number of bounces that occur before the laser passes through both pumpkins?

5. Jae is extremely scared of long lengths, particularly those longer than 241. Naturally, Albert terrifies him by drawing an octagon of side length 100 and all of its diagonals. How many of the diagonals scare Jae?

6. The candy corn problem continues! Charley needs to feed an uncountably infinite number of Charmanders, so he has made an ultra-large candy corn isosceles triangle $\triangle ABC, AB = AC$, which has been cut into $n > 1$ regions of equal area by segments parallel to its base. If the thickness of a region is the fraction of the altitude from A to BC contained in it, then Charley's new candy corn has 73 consecutive regions that do not contain point A or segment BC with a combined thickness of $\frac{1}{37}$. What is n?

7. On an 11×11 square grid, a headless horseman starts at the bottom left corner and wants to reach the top right corner by jumping in a knight's-move pattern; that is, two squares in one direction, then one square in a perpendicular direction. Unfortunately, there is a sinkhole starting in the middle of the grid! The sinkhole and the horseman take turns moving, with the sinkhole going first. On the sinkhole's turn, it picks a square that the horseman has not visited and is adjacent by an edge to a square that is currently a sinkhole, and turns it into a sinkhole. On the horseman's turn, he does a knight's-move to another square of the grid that is not a sinkhole. The horseman's goal is to reach the top right corner, and the sinkhole wins if the horseman cannot do this. Which side has a winning strategy?

8. A perfect right triangular slice of a pumpkin pie has integer side lengths and altitudes. What is the smallest possible area of this slice?

9. Ritvik is bored after a night of trick-or-treating and evaluates

$$P = (0^2 - 110)(1^2 - 109)(2^2 - 108) \cdots (109^2 - 1)(110^2 - 0).$$

What is the remainder of P when divided by 100?

10. You walk up to a gate in the graveyard. A sign tells you that the password is the number of positive integer factors of

$$N_{gate} = |26(49 - 83)^3 + 83(26 - 49)^3 + 49(83 - 26)^3|.$$

What is it?