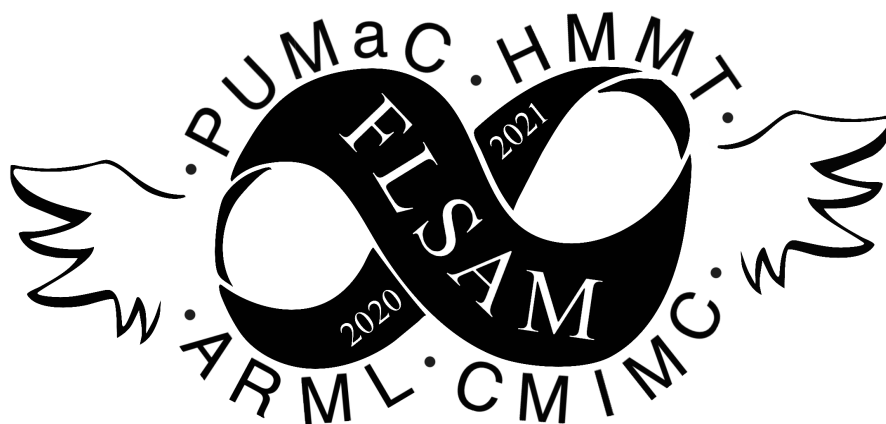


Florida Student Association of Mathematics



2020 Spooky Halloween Contest Individual Round

October 2020

Welcome to the **2020 FLSAM Spooky Halloween Contest**! This is the individual round, consisting of 10 problems in the subjects of algebra, geometry, number theory, and combinatorics. Each problem is worth 1 point, and problems are not necessarily in order of increasing difficulty.

*You will have **40 minutes** to complete the test. Good luck, and have fun!*

1. What is the largest prime factor of 168421?
2. You have a pumpkin patch, starting off with 0 pumpkins on day 0. Everyday after day 0, a prime number of pumpkins are in your patch. If the number of pumpkins is strictly increasing but each day cannot grow more than 10 new pumpkins, what is the maximum number of pumpkins that you can have by day 5?
3. Vismay is giving trick-or-treaters either a trick or a treat. Iris keeps track of his *spookiness*, which is a number starting from 0. Each time Vismay tricks, the spookiness increases by 5, and each time Vismay treats, the spookiness decreases by 3. After each of Vismay's actions, Iris computes the absolute value of Vismay's spookiness, and finds the sum of these values. What is the smallest possible sum that Iris obtains after Vismay does eight actions?
4. In regular hexagon $HAUNTS$ with side length 1, the midpoint of \overline{HU} is M and the midpoint of \overline{MN} is O . What is the length of OU ?
5. Points $PHANTOM$ are in the plane with $PHAN$ and $PTOM$ unit squares. Their region of intersection has area $\frac{1}{2}$; what is the length of HM ?
6. Karthik is attempting to summon the demon Exponentius, and his ritual includes writing a polynomial $P(x) = ax^2 + bx + c$ with coefficients in the set $\{0, 1, 2, \dots, 9\}$. For each positive integer power 2^k of 2, $P(2^k)$ is also a positive integer power of 2. How many polynomials with this property can he write?
7. Ritvik is doing Halloween decorations and wants to create n pumpkins, each with a positive integer, such that no matter how he numbers them he will always end up with a pair of distinct pumpkins such that their sum or difference has a remainder of 0 when divided by 2016. Find the minimum possible value of n .
8. Consider all pairs (a, b) of natural numbers such that the number expressed as a^ab^b ends in 98 zeros in base 10. If the product of ab is minimized, determine the value of $a + b$.
9. There are many, many spooky skeletons each with a different number of bones. Each skeleton has a number of bones that is relatively prime to 2021 and the number of skeletons are maximized. If there are S bones in total, find the sum of all distinct prime factors of S .
10. Charley is attempting to create candycorn for his infinite Charmander army. Each one is an isosceles triangle $\triangle ABC$, $AB = AC$, with points E, F on segment AB and G, H on segment AC such that segments EG, FH are parallel to BC and trisect the area of $\triangle ABC$. If the height from A is dropped to BC , what fraction of it is in the region between segment EG and segment FH ?